

Rule 110 objects and other collision-based constructions

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Abstract

The one-dimensional cellular automaton Rule 110 shows a very ample and diversified glider dynamics. The huge number of collision-based reactions presented in its evolution space are useful to implement some specific (conventional and unconventional) computable process, hence Rule 110 may be used to implement any desired simulation. Therefore there is necessity of defining some interesting objects as: solitons, eaters, black holes, flip-flops, fuses and more. For example, this work explains the construction of meta-gliders; for these constructions, we specify a regular language in Rule 110 to code in detail initial conditions with a required behavior. The paper depicts as well several experimental collision-based constructions.

1 Introduction

The present manuscript constitutes a continuation from our previous results explained at “Gliders in Rule 110” [6]. In this way, we continue utilizing Cook’s

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notation to identify gliders and we use as well our regular language [7] to code phase-based initial conditions.¹

Using the above conventions, the construction of objects in Rule 110 will be discussed and explained. The objects are formed by two or more collision-based gliders, where their multiple reactions yield the production of a specific object in the evolution space of Rule 110.

Applying this paradigm, some base objects will be constructed to show both their utility in procedures explored in our previous work and their possible potential applications. For example, the objects can be seen as components whose eventual synchronization realizes a particular task.

One special case in Rule 110 is the presence of soliton reactions [8]; they are relevant to preserve information and for conserving operators in the implementation of a cyclic tag system [3, 17]. We shall analyze this phenomenon in binary reactions.

The paper is organized in the following way: section 2 describes the construction of meta-gliders, section 3 explains the composition of large triangles, section 4 exposes the definition of solitons and section 5 studies the production of several Rule 110 objects. The codification of the regular expressions (using phases f_i) defining the whole set of constructions proposed in this work is given at a final appendix.

2 Constructing meta-gliders

Several gliders in Rule 110 are able to form meta-gliders by means of a careful synchronization of different reactions in the evolution space, as a large lattice in Rule 110.

With de Bruijn diagrams we can find distinct lattices with diverse tiles (see Figure 8 of [6]). But now the constructions are more complicated because we need to synchronize various reactions at the same time.

Two examples of meta-gliders are displayed in Figure 1. Figure 1 (a) shows a triple collision between A , D_1 and C_1 gliders. The A glider crosses as soliton and the collision produces the D_1 and C_1 gliders, something interesting is that the A glider helps to maintain the other collisions in a suitable phase adjusting each production.

Figure 1 (b) is more simple, it uses many glider gun's so to cancel B gliders by A gliders emitted periodically.

Now we show the construction of three meta-gliders with long period (figure 2). Each meta-glider has two evolutions. The first picture (left evolution) has an evolution with a small initial condition in order to determine the phenomenon with boundary properties, the following picture (right evolution) displayed the construction for a space of arbitrary size.

Our first construction is in Figure 2 (a) showing a reaction between a B glider and an \bar{E} glider which is almost a soliton, because A and B gliders are

¹An explanation of the regular language and electronic file is available from <http://uncomp.uwe.ac.uk/genaro/Rule110.html>

cancelled and the \bar{E} glider goes on with an extra B glider (left evolution). In the second picture (right evolution), the regular expression constructs a meta-glider with almost the same soliton effect. Here it is important to introduce a sequence of ether to establish a suitable distance for conserving the synchronization. The B glider may just interact in two ways with the \bar{E} glider for producing the same result, which facilitates the construction of this meta-glider using any phase in both gliders.

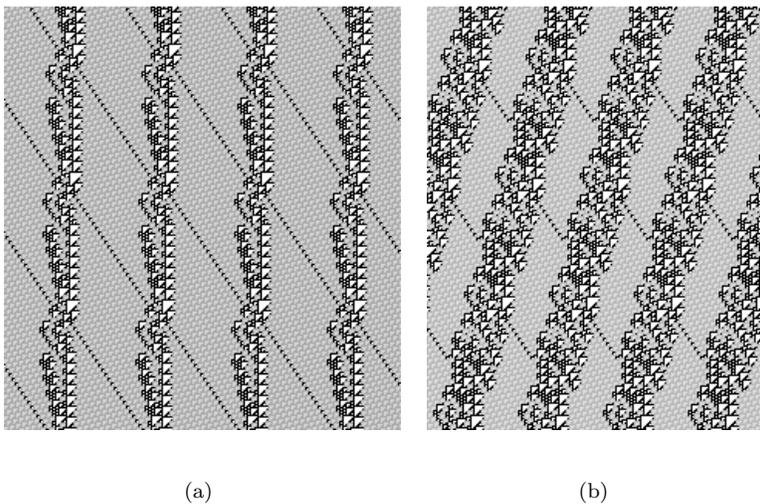
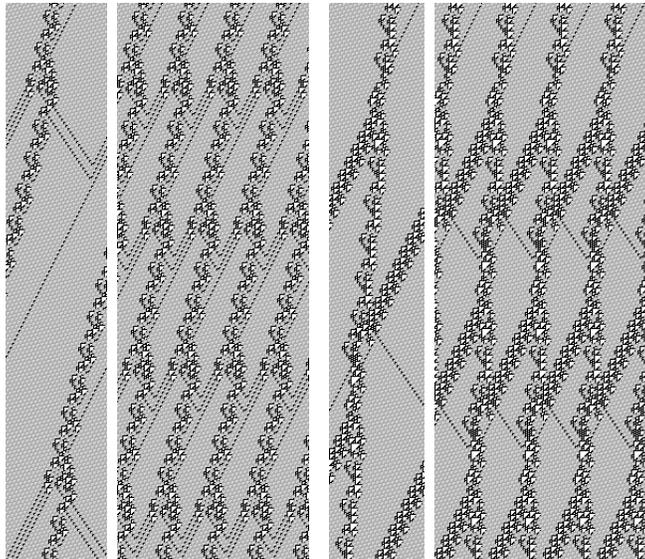


Figure 1: Meta-gliders in Rule 110.

The second construction in Figure 2 (b) shows a cycle defined by F and B gliders, producing again a \bar{B} glider and a F glider. Then the \bar{B} glider collides the new F glider yielding another B glider cancelled by an A glider. In this example it is more difficult to see these collisions but all of them are conserved, one can see a best reaction-structure giving more distance mod 4 (by tiles) or mod 14 (by cells) between F and B gliders.

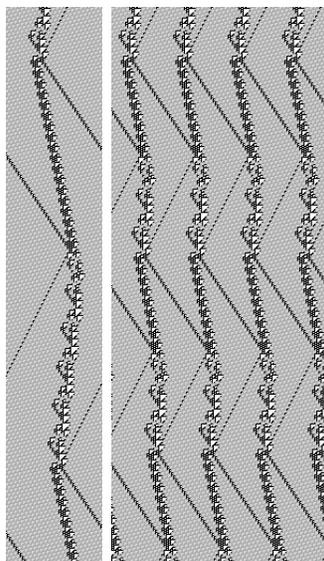
Our third example in Figure 2 (c) is a little bit different and is realized with A 's, B and F gliders. The phenomenon is particularly interesting because it can simulate a square-potential wall taking adequate initial conditions [12]. A particle bounces in the wall without crossing it, then having the correct phase and distance parameters, the particle crosses the wall simulating a “tunnel effect.” The B glider may cross one F glider at least in one collision, this phenomenon is obtained working with the distances among gliders for controlling the final result.

The production begins with a collision between a F and B gliders, producing a D_1 and a package of A^2 gliders, then these gliders collide regenerating the F and B gliders. In the meta-glider it is clear to see how the A 's and B gliders oscillate in their central parts, like a membrane configuration [9].



(a)

(b)



(c)

Figure 2: Meta-gliders with long period.

Another interesting example is displayed in Figure 3 illustrating a periodic or non-periodic meta-glider in its construction. The first picture (left evolution) depicts the periodic behavior with boundary properties and four reactions. The second picture (right evolution), depicts its non-periodic behavior with multiple reactions. This example requires a really laborious codification for the initial condition and it is a good example of synchronizing collisions for a number of gliders in the evolution space of Rule 110. Thus we have two parts: the first one by two separated A gliders and a package of A^4 gliders; the second is formed by a \bar{B} , a package of B^2 gliders and an E glider merged with an \bar{E} glider.

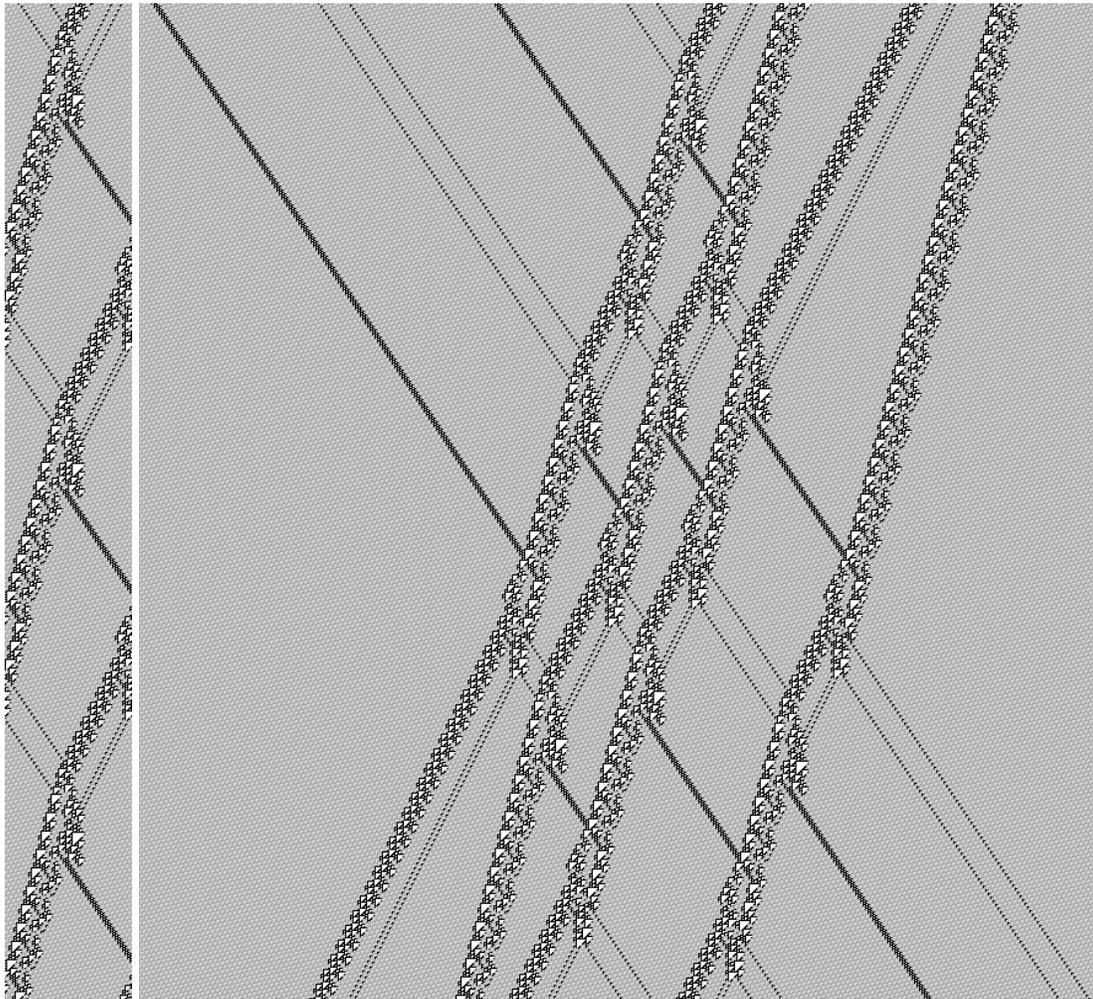


Figure 3: Periodic and non-periodic meta-glider

The production begins with the A and \bar{B} gliders which should produce two

merged \bar{E} gliders. But the second A glider coming from the left, transforms the \bar{E} into an E glider, and the B^2 gliders arriving from the right yield the A^4 and \bar{E} gliders, leaving the E merged with the \bar{E} glider. The package of A^4 gliders collide with the E and \bar{E} gliders returning to the sequence of \bar{B} , A , A and B^2 gliders, starting a new cycle of reactions.

We observe as well that the cycle can be interpreted as a soliton-reaction because the package of A^4 gliders crosses in each collision as two A gliders, conversely the two A gliders cross as a package of A^4 gliders (each with double period). On the other hand the \bar{B} and B^2 gliders cross like a soliton the E and \bar{E} gliders after each collision of double period, and the E and \bar{E} gliders return to the \bar{B} and B^2 gliders, i.e., the opposite case.

3 Constructing large triangles

McIntosh has appointed two relevant problems to Rule 110. The first is characterizing the covering of the evolution space with tiles and the second is to determine the largest tile produced by collisions in Rule 110 [11].

Brute-force computations looking for ancestors establish initial conditions to construct the T_{43} , T_{44} and T_{45} tiles, and a careful revision of the de Bruijn diagram for several generations determined that it is not possible construct tiles greater than T_{45} after 9 generations; this limit was established in August 2001 [11]. Later, at December 2002, Cook constructed tiles with size T_n for $27 \leq n \leq 33$ through non-periodic sequences in the initial conditions. Every sequence determines a particular tile and in some cases the change of one or two bits in the sequence produces an equal or smaller tile.

We have analyzed this problem an finding that tiles T_{24} , T_{25} , T_{26} , T_{28} , T_{29} y T_{30} can be generated by glider collisions. Tiles with a size smaller than 21 appears with more frequency in the evolution space and they easily rise from random initial conditions. Figure 4 presents a production for each tile.

In order to obtain a tile T_{26} we need a C_1 , a F , and an E^3 glider specified by an E and two B gliders, and finally a package of $6B$ gliders. In this case, as in other problems in Rule 110, the change of one structure in only one index induces a complete variation in the final production. For instance, if the C_1 glider is replaced by a C_2 or C_3 glider, the tile T_{26} is not composed; only in a very few cases this kind of change yields a larger tile. To bear a tile T_{28} we need four spaced C_1 gliders, an \bar{E} and a B glider for controlling the right chaotic region and generate the tile. In this case the absence of the B glider in the required phase determines the full disappearance of T_{28} , not even creating a closer tile.

For getting a tile T_{29} we require a package of A^5 gliders colliding with two joined F gliders, but a few steps before a B glider interacts with the second F and a G glider determines the right margin building the T_{29} .

The tile T_{29} was calculated in 276 generations and when the initial condition was formulated, an out-of-phase glider produced accidentally a tile T_{30} . Thus the sequence reproducing the T_{30} is the same that the one for the T_{29} ; just an extra

A glider at the left is demanded. Some variations of this expression may produce tiles T_{16} , T_{20} and T_{22} .

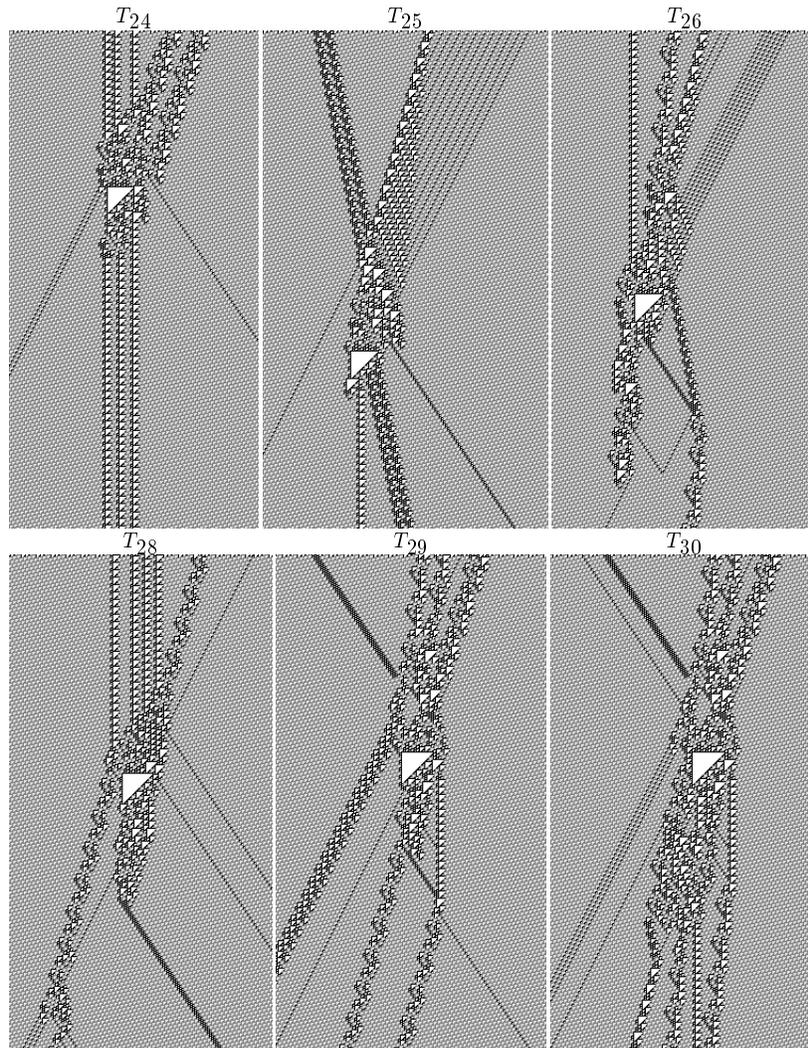


Figure 4: Producing large collision-based triangles.

However, we can say that in the formation of tiles T_{29} and T_{30} there is a non-proper collision, because there exists a B glider between a pair of slower F and G glider. In this way we must determine if there is a production providing the B and G glider for harmonizing a proper collision.

Open problems are the generation of tiles T_{27} , and T_{31}, \dots, T_{42} by means of collisions.

4 Solitons in Rule 110

Rule 110 can be able of simulating the soliton phenomenon in a natural way. This one was observed by a systematic analysis making all binary collisions between gliders [5].

Solitons and collisions in general are applied to realize conventional or unconventional computation transmitting information [2]. For example, Cook uses several soliton-reactions in Rule 110 for constructing the cyclic tag system [3].²

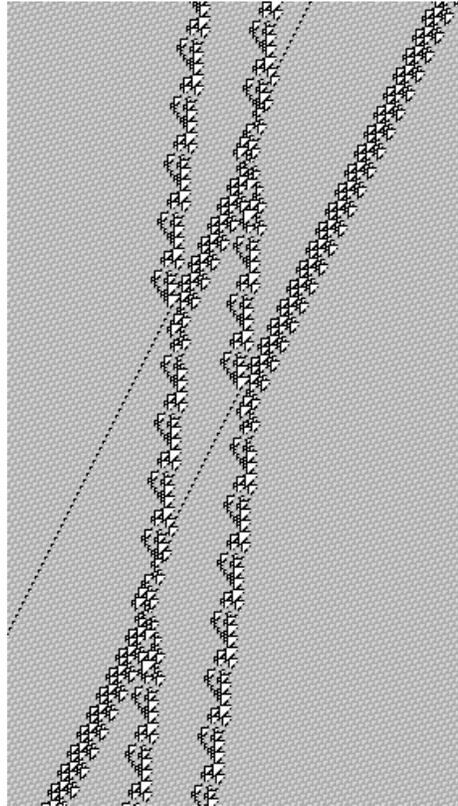


Figure 5: Pseudo-soliton with F , B and \bar{B} gliders

A soliton is characterized by two arbitrary constants determining speed and amplitude, a variable defines the soliton coordinate in a given time and the unitary vector determines its polarization (or phase). The special meaning of solitons is that in a certain way they determine the asymptotic state of an arbitrary solution. Thus, solitons are stable in the sense that small changes in its initial conditions yield small changes in its parameters [8].

²You can see a full simulation of the cyclic tag system in the evolution space of Rule 110 from <http://uncomp.uwe.ac.uk/genaro/Rule110.html> section “cyclic tag system.”

Different types of soliton-reactions can be constructed in Rule 110 [5]. Packages of solitons can be interpreted as meta-gliders, because we may have several different gliders interacting as solitons; although the synchronization is really complicated in many cases. All binary soliton-reactions between gliders are enumerated in our appendix and they will be discussed in a next paper.

In this subject we have a special case that we call pseudo-soliton originated by the reaction $F \leftarrow B = \bar{B} \wedge F$ and $F \leftarrow \bar{B} = B \wedge F$. Then B glider is transformed into a \bar{B} glider which even if conserves its speed, it does not have the same form. The transformation is possible because \bar{B} glider has 1/11 possibilities of returning into a B colliding against a F glider as Figure 5 shows.

But, we have a limitation in this example because is not possible to insert other F glider to the left and continue the pseudo-soliton-reaction, because the second \bar{B} cannot produce another B glider in any possible case, thus the restriction is that we have not periodic productions after of two solitons.

Let us remember that the models until now known in cellular automata theory, do not have any direct relation with solutions of non-integrated partial differential equations. Some important results trying to establish such a relation can be consulted by Kenneth Steiglitz [13, 8]. For example, multiple collisions of the soliton type between several gliders can help to construct computable systems, like the Manakov's model [8] or excitable models [1]. In the next section we discussed some applications of solitons for the cyclic tag system.

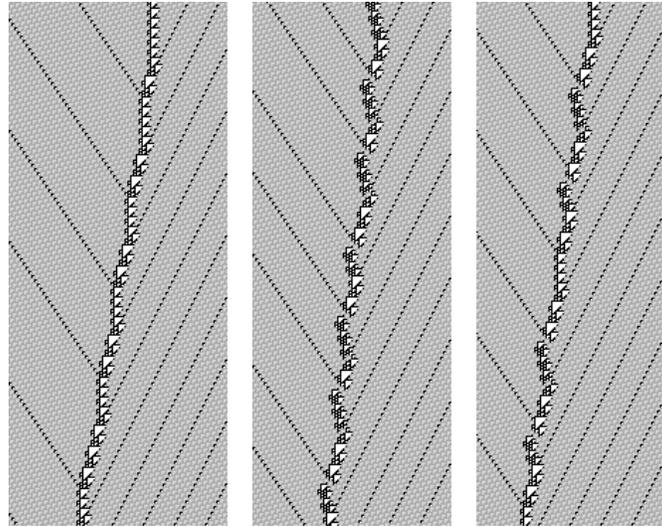
5 Rule 110 objects

Many years of research in Life have been dedicated for the search and construction of different objects, some of them simple and others very complicated [2, 4]. Several objects found by different investigators in a number of years allowed finally the specification of a Turing machine in Life [2] (by Paul Rendell). Therefore the exploration and definition of simple and complex objects will help to solve open problems in Rule 110. For example, implementing a Turing machine, self-reproducing components, intrinsic universal systems and an universal constructor between others.

The representation in the one-dimensional case is significantly different that in two or three dimension because we have an auxiliary two-dimensional evolution space. Nevertheless, the constructions are equally exhibited.

You can see an ingenious representation by Andrew Wuensche with animation of rings for the one-dimensional case applying the DDLAB system.³ All reactions like reflections, eaters, annihilations, solitons, black holes, fuses and more are totally identifiable (although it is necessary a previous experience in the operation of one-dimensional cellular automata).

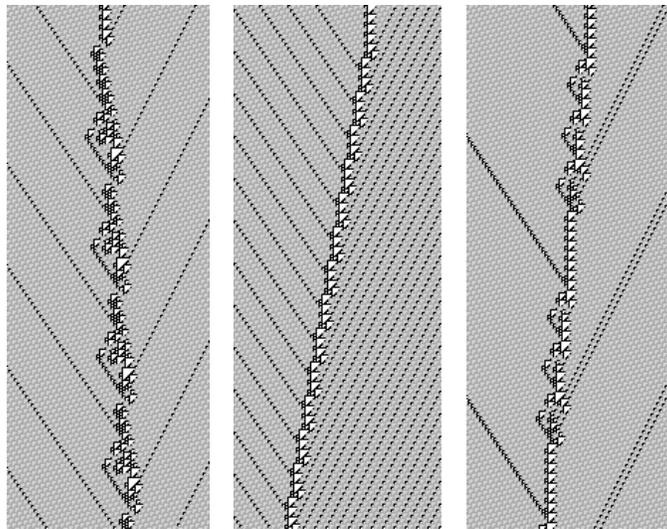
³<http://www.ddlab.org/>



(a)

(b)

(c)



(d)

(e)

(f)

Figure 6: Six black holes patterns in Rule 110.

5.1 Black holes

Now we present gliders which periodically absorb gliders, better known as black holes. In these constructions distances can vary, but the important thing is to conserve the collision absorbing gliders from left to right. Figure 6 show six examples, all of them absorbing only A and B gliders.

The first case in Figure 6 (a) illustrates the collision among C_3 and B gliders producing an E glider, the E receives an A glider returning it into a C_3 glider. The distance between A gliders is always $\text{mod } 3$ (according to the ether background) and for the B glider only a phase is necessary.

The second black hole in Figure 6 (b) is produced between a D_1 against a B glider yielding an E glider, this in turn collides against an A producing again the D_1 glider similar to the previous example. The difference is the C_3 glider instead of the D_1 .

The third case in Figure 6 (c) is a mixture of the two previous examples. This black hole oscillates in its central part between C_3 , E and D_1 gliders.

The fourth example in Figure 6 (d) is a more elaborated collision and we must take care about a suitable distance to obtain the desire construction. A C_2 glider collides against a B producing a D_1 glider. Next, an A collides the D_1 producing a small chaotic region that should produce a C_2 glider. But a B arriving from the right before time does not allow it and produces a D_1 glider, absorbing the A and B gliders beginning a new cycle of reactions.⁴

The fifth black hole in Figure 6 (e) is a collision among C_1 gliders against B gliders producing a C_2 that will be quickly transformed. In this case the arriving A eliminates the C_2 glider immediately returning to a C_1 . In a long way the sequence of collisions should be: $C_1 \leftarrow B = C_2 \wedge A \rightarrow C_2 = C_1$.

The last black hole in Figure 6 (f) absorbs pairs of A and B gliders. A package of A^2 gliders colliding against a C_2 produces a F glider, then a package of B^2 collides with the F returning to the C_2 .

Up to now, we have not found other black holes absorbing another type of gliders; suggesting that they will be more rare or sophisticated with a very long period.

5.2 Eaters

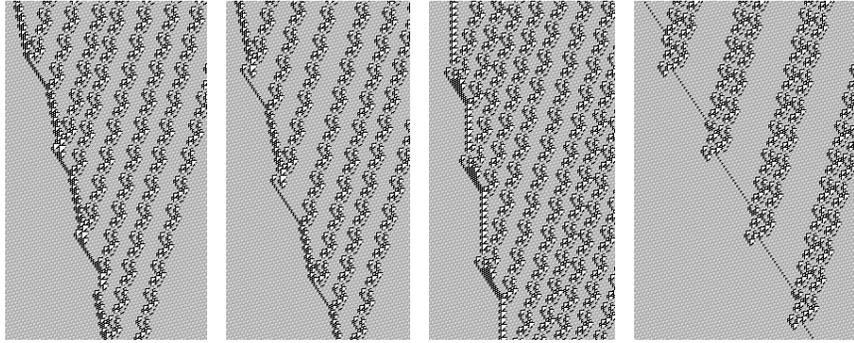
Rule 110 has gliders which eat other gliders without leaving any rest. For example, at the cyclic tag system, we can find an object eating gliders.⁵

In the first case a D_1 and a package of A^3 gliders delete E gliders in each collision (figure 7 (a)). The same phenomenon can be reproduced with a D_2 and A^2 gliders as it is seen in Figure 7 (b).

The third case deletes as well \bar{E} gliders but now with a C_1 and a package of A^5 gliders (Figure 7 (c)). The fourth case in Figure 7 (d) is between a pair of

⁴The original cycle must be among C_2 and D_1 gliders, but with a B glider we avoid the existence of C_2 , thus the original production is: $C_2 \leftarrow B = D_1$ and $A \rightarrow D_1 = C_2$.

⁵See <http://uncomp.uwe.ac.uk/genaro/Rule110.html>, section cyclic tag system or [3, 17].

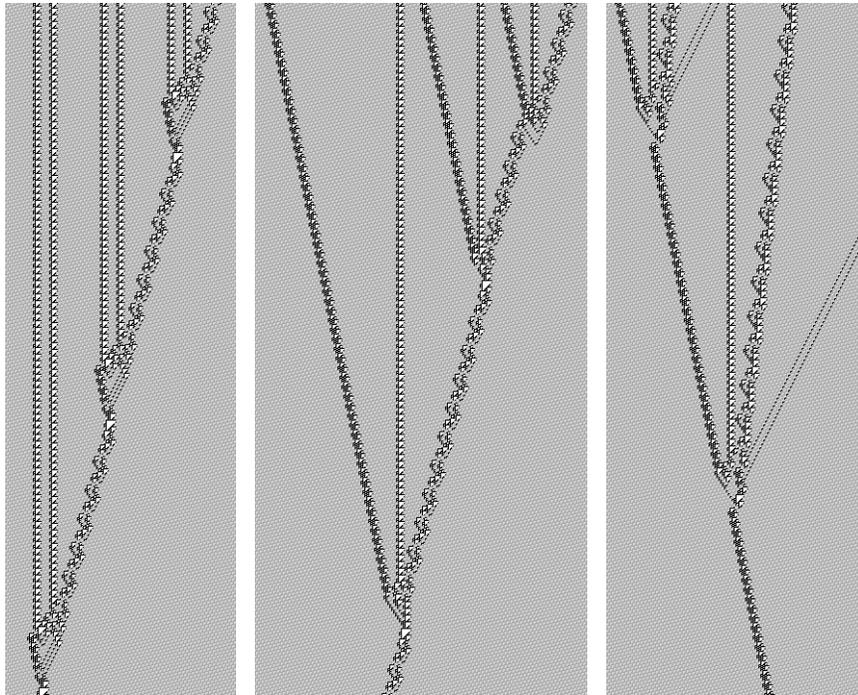


(a)

(b)

(c)

(d)



(e)

(f)

(g)

Figure 7: Seven eater patterns in Rule 110.

\bar{E} 's and an A glider, this one deletes both \bar{E} gliders and continues its trajectory with a delay.

Our fifth example in Figure 7 (e) eliminates a pair of C gliders, they are a C_3 and a C_2 hitting an \bar{E} glider. The first collision between C_3 and \bar{E} generates five B gliders (that will be deleted quickly), besides the C_2 produces a D_1 after colliding against the B^3 gliders returning into the \bar{E} . This production is repeated for each pair of C 's forming an \bar{E} with a minimal distance of $3e$ configurations (mod 42) between C gliders.

The sixth eater in Figure 7 (f) is significantly different from the previous ones because there is an interval needed to synchronize a triple collision between D_1 , C_2 and \bar{E} gliders (p^+ , p^0 and p^- slopes, see [6]) with an interval constant growing in an exponential factor of $3^n e$ (for all $n > 0$) between each D_1 and C_2 gliders. Then they are eliminated by an \bar{E} glider at each collision. The evolution shows three different collisions, nevertheless, the final reaction is the same one.

The last eater in Figure 7 (g) is even more complicated. The elimination of gliders was yielded by a glider of positive slope p^+ . Thus a D_1 eats a C_3 with a F and two B gliders, showing a four-tuple reaction!

Some collisions among gliders cause an *annihilation* reaction [5]. The simplest case is the collision between A against B gliders where both disappear. The same phenomenon may be observed in one of the three possible collisions between an A against a \bar{B} glider. Another example is a pair of C_2 's eliminated by a B or a \bar{B} glider.

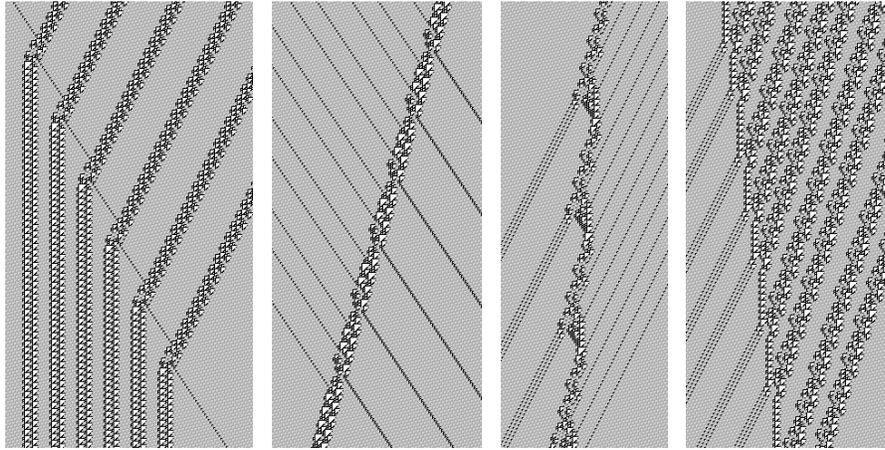
5.3 Fuses

Rule 110 has fuse configurations as well, they are produced by consecutive collisions and some of them are very interesting as Figure 8 illustrates. One can extract several examples of fuse configurations from de Bruijn diagrams for 10 generations (see full patterns in Figure 8 from [6]). The phenomenon is generated when a periodic sequence into a cycle change to another cycle (or more), then the evolution space is composed by two different patterns.

The first example in Figure 8 (a) is constructed between an A against a \bar{B} glider. The collision produces two merged C_3 gliders; the important thing is that the A gliders survival and continue, thus we can always repeat this process (describing a path among two or more cycles into de Bruijn diagram; this path has a direct correspondence with a regular expression of our regular language L_{R110} [7]).

The second fuse in Figure 8 (b) is with an A glider colliding with a package of two E 's producing a D_1 glider and a very small chaotic region. Later the intervention of other A glider in the right contact point allows to return to the two E gliders. The A glider arrives at two different phases with a constant distance of $2e$ and e configurations, producing in the process a package of A^2 gliders merged by the collision among the D_1 and E gliders.

The third fuse figure 8 (c) is with a \bar{E} glider and B gliders arriving from the right. The first collision produces a package of B^3 gliders. The second B glider produces a C_1 glider hitting at the same time against a package of A^4

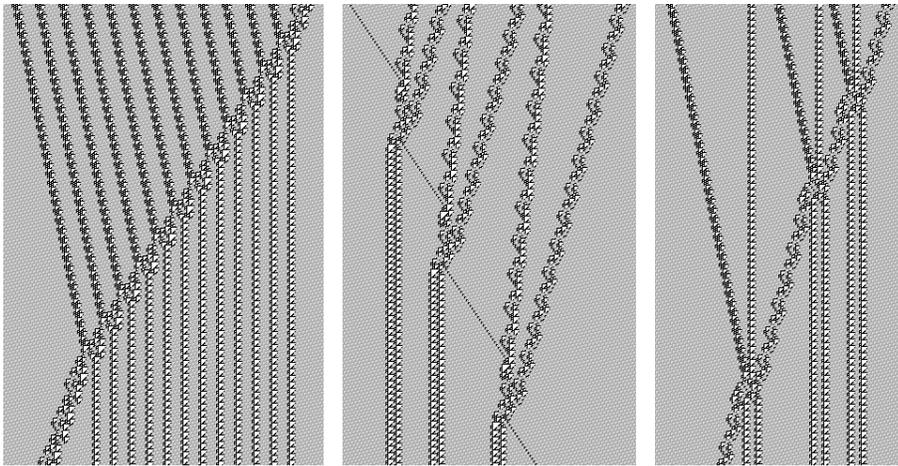


(a)

(b)

(c)

(d)



(e)

(f)

(g)

Figure 8: Seven fuse patterns in Rule 110.

gliders and with the third B glider we return to the original \bar{E} glider. The fourth example in Figure 8 (d) also produces packages of B^3 but now with C_2 and G gliders.

The fifth case in Figure 8 (e) begins with a D_1 against a G glider. The result are a G and a C_3 glider. As the G glider is preserved after the collision, we just added more D_1 gliders to obtain this same change. The distance between each D_1 glider is the minimum needed to conserve the structure. Nevertheless, it is possible to extend the distance to obtain a clearer presentation of G .

The sixth fuse in Figure 8 (f) is more complicated because we need a triple reaction with three different slopes, producing a pair of merged C_3 's as in the first fuse. But in this construction, the intervals among F and \bar{E} gliders grow by an exponential factor of $2^n e$ (for all $n \geq 0$) for each collision. The reactions may be stopped with the same gliders changing the phase of the last \bar{E} without producing other gliders.

The seventh fuse in Figure 8 (g) is another triple collision among D_1 , C_2 and \bar{E} gliders producing a C_2 with a C_3 glider. Again, the important problem is to adjust the interval among the D_1 and the C_2 glider.

5.4 Flip-flop

A flip-flop configuration in Life is a stationary periodic structure oscillating with period two (period three or superior are known as blinkers [14]). This patterns represent as well the value of signals in the design of circuits.

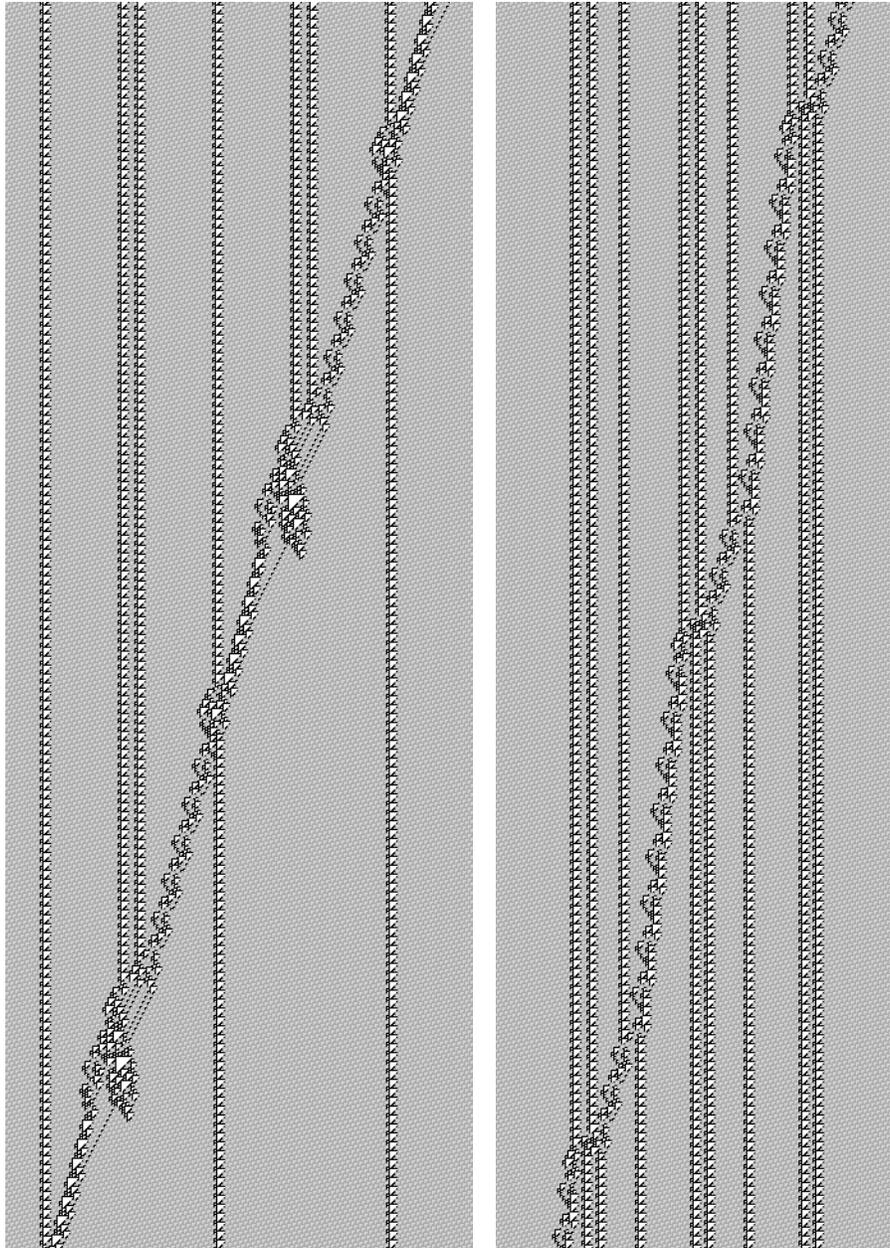
In order to represent a flip-flop pattern in Rule 110 we need several interacting gliders, i.e., the implementation is not realized by a unique glider (as the case of Life with a line of three live cells). For example, Figure 9 displays evolutions that may be interpreted as flip-flop configurations.

In the first example of Figure 9 (a) we initiated with an E^2 colliding a C_2 producing an \bar{E} glider. In order to return into E^2 the \bar{E} collides against two C_2 gliders. In this production there is an exceeding C_1 . Thus the flip-flop construction is between $E^2 \rightarrow \bar{E}$ gliders; although the device is not reusable.

The second example in Figure 9 (b) illustrates the oscillation between \bar{E} and F gliders. To obtain the result we have the \bar{E} glider colliding with two C_3 producing a F with two stationary C_1 and C_2 gliders. The C_1 hit the previous F producing a C_2 glider. In order to return to \bar{E} the F must collide against a C_1 glider. Again, the device is not reusable to future reactions.

5.5 Applying some useful Rule 110 objects

In Rule 110 we have some periodic objects that can be useful to construct a desired process; nevertheless, a search for obtaining additional valuable objects must be done. Rule 110 offers a wide variety of these objects. An immediate application of Rule 110 objects was realized by Cook in several stages for developing the cyclic tag system [3]. Next we displayed some of them illustrating their constructions.



(a)

(b)

Figure 9: Two flip-flop patterns in Rule 110.

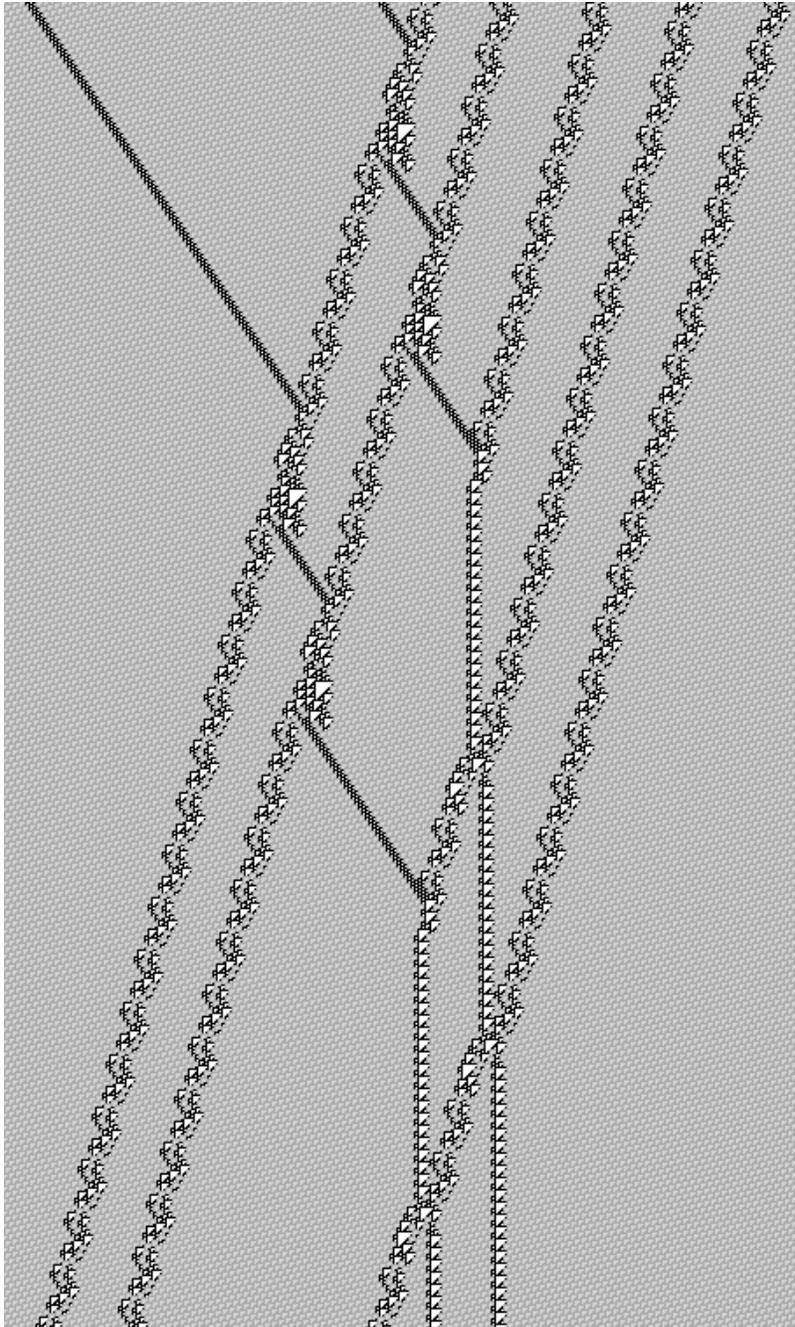


Figure 10: Some Rule 110 objects of the cyclic tag system representing operators and data.

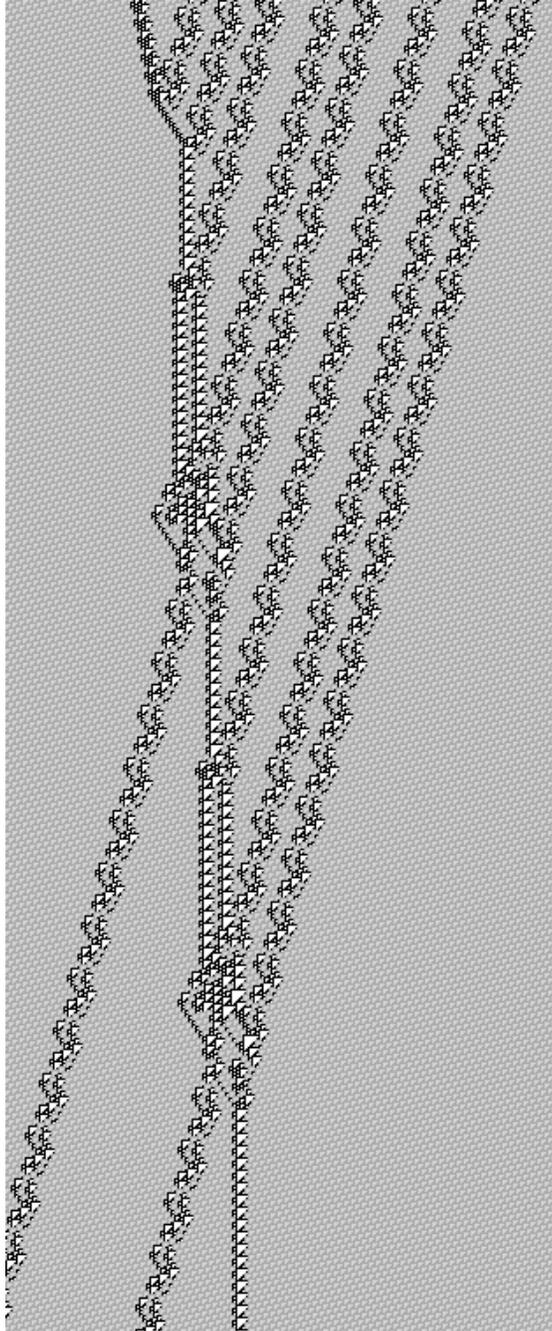


Figure 11: Some Rule 110 objects of the cyclic tag system deleting signals and preparing new data.

Figure 10 illustrates two important reactions of the cyclic tag system. There is a reaction with A^4 gliders which are preserved as solitons crossing an \bar{E} (other soliton); they maybe will not be immediately taking part of the system and will be subsequently used. A second reaction packing A^4 gliders transform each \bar{E} into C_2 gliders, they are interpreted like data on the tape. The central part is very important to simulate the cyclic tag system in Rule 110 because it represents the interaction among operators and data arriving from the right transformed previously by blocks of E 's.

The second evolution (Figure 11), packages \bar{E} gliders periodically transformed by a C_3 and a pair of C_1 gliders. The blocks of \bar{E} gliders are converted into other data blocks (consisting of only four \bar{E} gliders) representing values in the tape on colliding with the package of A^4 gliders. All reactions begin with a D_1 arriving from left.

In both examples we can identify collisions controlled in different stages. Local or general synchronization is determined by distances and phases. Although the implementation in Rule 110 is impressive, many details must be discussed to suitably reproduce the cyclic tag system (you can analyze the full simulation and multiple details for each stage, see footnote 2).

In Figure 11 the first evolution is another example of the cyclic tag system illustrating that a simple construction can be combined with others to elaborate a specific operation. It starts with a group of four C_2 's representing data and they are erased by an \bar{E} yielding two A 's used in other collisions. In the figure, two blocks of C_2 gliders are erased.

The second evolution constructs a cycle between E and a pair of C_2 gliders, producing periodically one A glider. The third evolution is similar but here there are two different C 's gliders for returning into an \bar{E} in each collision.

The last example is constructed with two previous knowing reactions. Figure 12 shows a set of collisions that can be interpreted as a logic NOT gate; although the device cannot be employed again.

To modify data on the type with 0's and 1's, we utilize the collision of two pairs of \bar{E} 's and an A glider. Thus a collision with the A deletes a pair of \bar{E} 's representing a $1 \rightarrow 0$ transformation other possibility is that the pair of \bar{E} 's crosses as a soliton indicating a $0 \rightarrow 1$ conversion.

6 Conclusions

Rule 110 have an incredible number of reactions into the evolution space. We can think about an infinity number of collisions in two ways: with packages of gliders ($g_1 \rightarrow ng_2$) or with extensible gliders ($g_1 \rightarrow g_2^n$). The diversity of reactions allows to investigate the simulation of several complicated process, for instance conventional or unconventional computing procedures [1], solitons (particle machines) [8], self-reproduction [15] and artificial life [14]. In this sense, we have developed an useful computational tool for specifying any reaction by coding initial conditions with regular expressions [7].

De Bruijn diagrams have been analyzed [10, 16] to calculate some simple

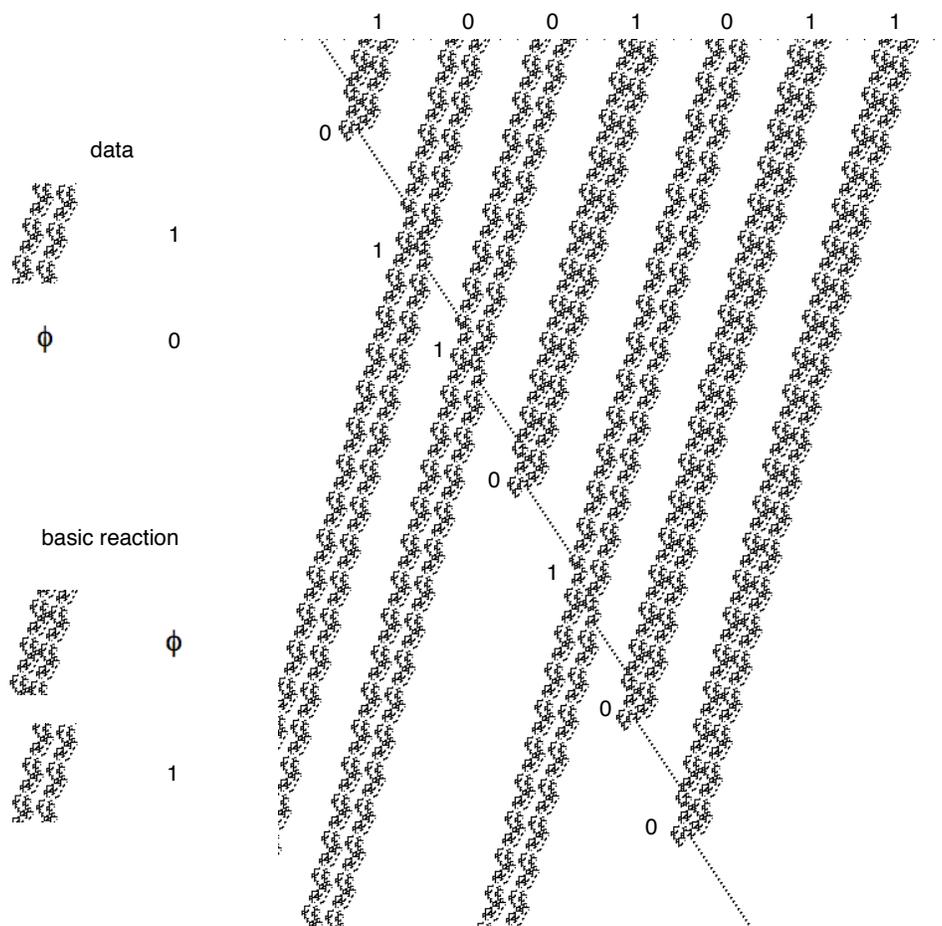


Figure 12: Simulating a simple NOT gate with Rule 110.

objects, but eventually a new way implementing specialized searches must be formulated. Another further work is to investigate the potential applications of the Rule 110 objects on unconventional computing.

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A Appendix

A.1 Table 1 – Meta-gliders

Relation of productions constructing meta gliders.

1. $\{A(f_{3-1})-3e-A(f_{2-1})-3e-A(f_{1-1})\}^*-e-D_1(A,f_{1-1})-C_2(A,f_{1-1})-\{e-A(f_{1-1})-e-D_1(A,f_{1-1})-C_2(A,f_{1-1})\}^*$
2. $\{\text{gun}(A,f_{1-1})-e\}^*$
3. (a) $\bar{E}(A,f_{1-1})-e-B(f_{1-1})$
 (b) $\{\bar{E}(A,f_{1-1})-B(f_{1-1})-e\}^*$
4. (a) $F(A,f_{1-1})-e-B(f_{1-1})$
 (b) $\{F(A,f_{1-1})-e-B(f_{1-1})-e\}^*$
5. (a) $F(H,f_{1-1})-e-B(f_{1-1})$
 (b) $\{F(H,f_{1-1})-e-B(f_{1-1})-e\}^*$
6. (a) $A(f_{3-1})-e-A(f_{1-1})-e-\bar{B}(C,f_{1-1})-e-2B(f_{4-1})$
 (b) $A^4(f_{3-1})-8e-A(f_{3-1})-e-A(f_{1-1})-8e-A^4(f_{1-1})-7e-A(f_{1-1})-e-A(f_{2-1})-2e-\bar{B}(A,f_{1-1})-e-B(f_{1-1})-B(f_{4-1})-E(B,f_{2-1})-\bar{E}(G,f_{1-1})-5e-\bar{B}(C,f_{1-1})-e-B(f_{1-1})-B(f_{4-1})-e-E(A,f_{2-1})-\bar{E}(A,f_{4-1})$

A.2 Table 2 – Solitons

Relation of productions generating all the possible solitons by binary collisions.

Solitons

1. $A \leftrightarrow \bar{E}$ gliders
 (a) $A(f_{1-1})-e-\bar{E}(C,f_{1-1})$

- (b) $A(f_{1-1})-e-\bar{E}(D,f_{1-1})$
- (c) $A(f_{1-1})-e-\bar{E}(E,f_{1-1})$
- (d) $A(f_{1-1})-e-\bar{E}(H,f_{1-1})$
- 2. $A \leftrightarrow G$ gliders
 - (a) $A(f_{1-1})-e-G(A2,f_{1-1})$
- 3. $C_1 \leftarrow \bar{E}$ and $C_1 \leftarrow F$ gliders
 - (a) $C_1(A,f_{1-1})-e-\bar{E}r(A,f_{1-1})$
 - (b) $C_1(A,f_{1-1})-e-\bar{E}(D,f_{1-1})$
 - (c) $C_1(A,f_{1-1})-e-F(A,f_{1-1})$
- 4. $C_2 \leftarrow \bar{E}$ and $C_2 \leftarrow F$ gliders
 - (a) $C_2(A,f_{1-1})-e-\bar{E}(A,f_{1-1})$
 - (b) $C_2(A,f_{1-1})-e-F(B,f_{1-1})$
- 5. $F \leftrightarrow B$ gliders
 - (a) $F(A2,f_{1-1})-e-B(f_{1-1})$
- 6. $F \leftrightarrow \bar{E}$ gliders
 - (a) $F(A,f_{1-1})-e-\bar{E}(A,f_{1-1})$
 - (b) $F(A,f_{1-1})-e-\bar{E}(C,f_{1-1})$
 - (c) $F(A,f_{1-1})-e-\bar{E}(D,f_{1-1})$
 - (d) $F(A,f_{1-1})-e-\bar{E}(E,f_{1-1})$
 - (e) $F(G,f_{1-1})-e-\bar{E}(A,f_{1-1})$
 - (f) $F(G,f_{1-1})-e-\bar{E}(B,f_{1-1})$
 - (g) $F(G,f_{1-1})-e-\bar{E}(H,f_{1-1})$

Pseudo-soliton

- 1. $F \leftarrow B \leftarrow \bar{B}$ gliders
 - (a) $F(G,f_{3-1})-2e-F(A,f_{1-1})-e-B(f_{1-1})-5e-\bar{B}(B,f_{4-1})$

A.3 Table 3 – Large triangles

Relation of productions yielding large tiles by collisions. Tiles $T_{n < 20}$ are found in the list of all the binary collisions between gliders [5].

$$T_{20}: D_1(A,f_{1-1})-e-C_2(A,f_{1-1})-e-\bar{E}(C,f_{1-1})-2e-2B(f_{3-1})$$

$$T_{21}: A(f_{3-1})-4e-D_2(C,f_{1-1})-C_2(B,f_{1-1})-2e-B(f_{1-1})-4B(f_{2-1})$$

- T_{22} : $D_1(A, f_{1-1}) - e - C_2(A, f_{1-1}) - e - \bar{E}(C, f_{1-1}) - 2e - 4B(f_{3-1})$
- T_{23} : $D_2(B, f_{2-1}) - D_2(A, f_{4-1}) - 8e - E(B, f_{1-1}) - 10B(f_{4-1})$
- T_{24} : $C_3(B, f_{1-1}) - C_2(A, f_{1-1}) - C_2(A, f_{1-1}) - e - G(A, f_{1-1}) - G(C_2, f_{1-1})$
- T_{25} : $D_2(B, f_{2-1}) - D_2(A, f_{4-1}) - 5e - E(B, f_{1-1}) - 11B(f_{1-1})$
- T_{26} : $C_1(A, f_{1-1}) - 2e - F(A, f_{1-1}) - e - E(D, f_{1-1}) - 2B(f_{1-1}) - 2e - 6B(f_{4-1})$
- T_{28} : $C_1(B, f_{1-1}) - C_1(A, f_{4-1}) - C_1(A, f_{1-1}) - C_1(B, f_{1-1}) - e - \bar{E}(D, f_{1-1}) - 2e - B(f_{3-1})$
- T_{29} : $A^5(f_{1-1}) - 6e - F(B, f_{1-1}) - F(G, f_{1-1}) - B(f_{4-1}) - G(F, f_{4-1})$
- T_{30} : $A(f_{1-1}) - e - A^5(f_{1-1}) - 6e - F(B, f_{1-1}) - F(G, f_{1-1}) - B(f_{4-1}) - G(F, f_{4-1})$

A.4 Table 4 – Objects

Relation of productions specifying Rule 110 objects.

Black holes

1. $\{A(f_{2-1}) - 3e - A(f_{2-1}) - 3e - A(f_{3-1}) - 3e - A(f_{1-1}) - 3e - A(f_{2-1}) - 3e - A(f_{1-1}) - 3e\}^* - C_3(A, f_{2-1}) - B(f_{3-1})^*$
2. $\{A(f_{2-1}) - 2e - A(f_{3-1}) - 2e - A(f_{1-1}) - 2e\}^* - D_1(A, f_{1-1}) - \{e - B(f_{1-1})\}^*$
3. $\{A(f_{1-1}) - 3e\}^* - C_3(f_{1-1}) - B(f_{3-1})^*$
4. $\{A(f_{1-1}) - 3e\}^* - C_2(A, f_{1-1}) - B(f_{1-1}) - e - \{2e - B(f_{1-1}) - 2e - B(f_{3-1})\}^*$
5. $\{A(f_{1-1}) - e\}^* - e - C_1(A, f_{2-1}) - \{B(f_{2-1})\}^*$
6. $\{8e - A^2(f_{1-1})\}^* - e - C_2(A, f_{1-1}) - 3e - B^2(f_{4-1}) - 3e - B^2(f_{3-1}) - 4e - B^2(f_{2-1})$

Eaters

1. $D_1(B, f_{2-1}) - e - \bar{E}(A, f_{1-1}) - \bar{E}(A, f_{3-1}) - \bar{E}(B, f_{1-1}) - \bar{E}(A, f_{1-1}) - \bar{E}(A, f_{3-1}) - \bar{E}(B, f_{1-1}) - \bar{E}(A, f_{3-1}) - \bar{E}(A, f_{2-1})$
2. $D_2(A, f_{1-1}) - \bar{E}(A, f_{1-1}) - \bar{E}(C, f_{3-1}) - \bar{E}(A, f_{3-1}) - \{\bar{E}(B, f_{3-1}) - \bar{E}(A, f_{2-1})\}^*$
3. $C_1(A, f_{1-1}) - e - \{\bar{E}(C, f_{1-1}) - \bar{E}(D, f_{1-1}) - \bar{E}(B, f_{1-1}) - \bar{E}(C, f_{1-1}) - \bar{E}(E, f_{1-1}) - \bar{E}(D, f_{3-1})\}^*$
4. $A(f_{1-1}) - \{2e - \bar{E}(A, f_{1-1}) - \bar{E}(G, f_{4-1})\}^*$
5. $\{3e - C_2(A, f_{1-1}) - C_3(A, f_{1-1})\}^* - e - \bar{E}(B, f_{1-1})$
6. $D_1(A, f_{1-1}) - 9e - C_2(A, f_{1-1}) - D_1(B, f_{2-1}) - 3e - C_2(A, f_{1-1}) - D_1(B, f_{2-1}) - e - C_2(A, f_{1-1}) - e - \bar{E}(A, f_{2-1})$
7. $D_1(A, f_{3-1}) - e - C_3(A, f_{2-1}) - F(A, f_{1-1}) - 2e - 2B(f_{1-1}) - C_3(B, f_{1-1}) - 3e - F(D, f_{3-1}) - 13e - 2B(f_{4-1})$

Fuses

1. $A(f_{1-1})-\{3e-\bar{B}(A,f_{1-1})-3e-\bar{B}(B,f_{1-1})-3e-\bar{B}(C,f_{1-1})\}^*$
2. $\{A(f_{3-1})-2e-A(f_{1-1})-e\}^*-E(C,f_{3-1})-E(D,f_{4-1})$
3. $\bar{E}(A,f_{1-1})-\{B(f_{3-1})-B(f_{2-1})-e-B(f_{1-1})-e-B(f_{4-1})\}^*$
4. $C_2(A,f_{1-1})-G(F,f_{1-1})-G(C,f_{2-1})-G(E,f_{4-1})-G(A,f_{2-1})-G(A_2,f_{1-1})-G(A,f_{1-1})-G(E,f_{2-1})-G(B,f_{3-1})-G(E,f_{1-1})$
5. $\{D_1(A,f_{1-1})-e\}^*-G(H,f_{1-1})$
6. $A(f_{1-1})-4e-F(H,f_{1-1})-e-\bar{E}(G,f_{3-1})-F(A,f_{1-1})-2e-\bar{E}(C,f_{3-1})-e-F(E,f_{1-1})-4e-\bar{E}(H,f_{1-1})$
7. $D_1(A,f_{3-1})-5e-C_2(A,f_{1-1})-D_1(B,f_{3-1})-2e-C_2(B,f_{1-1})-D_1(A,f_{1-1})-e-C_2(A,f_{1-1})-e-\bar{E}(A,f_{1-1})$

Flip-flop

1. $\{3e-C_2(B,f_{1-1})-C_2(A,f_{4-1})-3e-C_2(A,f_{1-1})\}^*-e-E(A,f_{1-1})-B(f_{1-1})$
2. $\{e-C_1(A,f_{1-1})-2e-C_1(B,f_{3-1})-C_3(A,f_{1-1})\}^*-e-\bar{E}(A,f_{1-1})$

Computational objects

1. $A^4(f_{3-1})-13e-A^4(f_{2-1})-13e-A^4(f_{1-1})-e-\bar{E}(B,f_{1-1})-e-\bar{E}(D,f_{3-1})-2e-\bar{E}(C,f_{1-1})-2e-\bar{E}(H,f_{2-1})-2e-\bar{E}(E,f_{1-1})-2e-\bar{E}(C,f_{1-1})$
2. $D_1(C,f_{3-1})-e-\bar{E}(B,f_{1-1})-\bar{E}(C,f_{1-1})-\bar{E}(D,f_{1-1})-e-\bar{E}(B,f_{1-1})-\bar{E}(D,f_{1-1})-e-\bar{E}(B,f_{1-1})-e-\bar{E}(H,f_{1-1})-\bar{E}(B,f_{1-1})-e-\bar{E}(D,f_{1-1})-e-\bar{E}(B,f_{1-1})-\bar{E}(D,f_{1-1})-e-\bar{E}(B,f_{1-1})-e-\bar{E}(H,f_{1-1})$
3. $\{4e-C_2(A,f_{1-1})-e-C_2(B,f_{1-1})-e-C_2(A,f_{1-1})-e-C_2(A,f_{1-1})\}^*-e-\bar{E}(C,f_{1-1})$
4. $\{2e-C_2(A,f_{1-1})-e-C_2(B,f_{1-1})\}^*-e-E(A,f_{1-1})$
5. $\{3e-C_2(B,f_{1-1})-C_1(A,f_{1-1})\}^*-e-\bar{E}(B,f_{1-1})$
6. $A(f_{1-1})-2e-\bar{E}(A,f_{1-1})-\bar{E}(G,f_{4-1})-2e-\bar{E}(A,f_{1-1})-\bar{E}(H,f_{1-1})-2e-\bar{E}(H,f_{1-1})-\bar{E}(G,f_{1-1})-2e-\bar{E}(D,f_{1-1})-\bar{E}(B,f_{4-1})-2e-\bar{E}(D,f_{1-1})-\bar{E}(C,f_{1-1})-2e-\bar{E}(B,f_{1-1})-\bar{E}(H,f_{4-1})-2e-\bar{E}(A,f_{1-1})-\bar{E}(G,f_{4-1})$