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1/f Noise in the Computation Process by Rule 110

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An elementary cellular automaton rule 110 supports universal computation by emulating cyclic tag system and its evolution starting from random initial configuration exhibits $1/f$ noise. In this research we investigate the power spectra of the computation process of rule 110 emulating cyclic tag system. As a result, $1/f$-type power spectra are observed in the most actively interacting area among the whole array, while in the less active area the power spectra exhibit Lorentzian, Brownian or periodic types. These results suggest a possibility that the dynamics accompanied with $1/f$ noise and the one capable of performing computation overlap each other in cellular automaton rule space.

Keywords: Rule 110, computational universality, cyclic tag system, 1/f noise, Brownian noise, white noise

1 INTRODUCTION

It is known that elementary cellular automaton (ECA), namely one-dimensional and two-state, three-neighbour CA rule 110 is computationally universal [1] that means any algorithms can be performed by setting appropriate initial conditions. On the other hand, the evolution of rule 110 starting from random configuration exhibits $1/f$ noise [2]. $1/f$ noise is a random process whose spectrum as a function of the frequency f behaves like

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$1/f^{\beta}$ with $\beta \approx 1$ at low frequencies. $1/f$ noise has been observed in various phenomena such as the voltages or currents of vacuum tubes, diodes, and transistors or the frequency of quartz crystal oscillators, but a general explanation of these phenomena has not yet been forthcoming [3]. Moreover the Game of Life (LIFE), a two-dimensional and two-state, nine-neighbour outer semi-totalistic CA, supports universal computation [4] and its evolution starting from random configuration exhibits $1/f$-type spectrum [5]. These results suggest that there might be a relationship between computational universality and $1/f$ noise in CAs although further research is necessary to confirm this hypothesis.

Let us attract attention to the fact that $1/f$ noise in ECA rule 110 and LIFE has been observed in the evolution starting from random initial configuration in previous works [2, 5]. However, the power spectra of the computation process of ECA rule 110 have not been investigated yet. The evolution starting from random configuration is solely influenced by the transition function, whereas the evolution performing computation is produced by the synergy between the transition function and the elaborately designed configuration that prescribes the procedure. Therefore we can guess that the latter power spectra capture the character of computation more appropriately from the viewpoint of dynamical systems than the former one. In this paper we study the computation process of ECA rule 110 by means of power spectral analysis. In section 2 we make a brief explanation of the computation process of rule 110 emulating a cyclic tag system (CTS). The results of spectral analysis are shown in section 3. Finally, we discuss the implication of the results in section 4.

2 CYCLIC TAG SYSTEM EMULATED BY RULE 110

The transition function of ECA rule 110 is given by following scheme:

$$\frac{111}{0}\ \frac{110}{1}\ \frac{101}{1}\ \frac{100}{0}\ \frac{011}{1}\ \frac{010}{1}\ \frac{001}{1}\ \frac{000}{0}.$$

The upper line represents the state of the neighbourhood and the lower line specifies the state of the center cell at the next time step. Cook proved the computational universality of rule 110 by showing its capability to emulate CTS [1] as an evolution on its array.

CTS is a variant of tag system [6] that is proved to be computationally universal. In CTS the alphabet consists of only two symbols, ‘0’ and ‘1’ and the deletion number that is the number of the leftmost symbols deleted at each step is fixed at one. If the leftmost symbol is ‘1’, then the CTS appends the

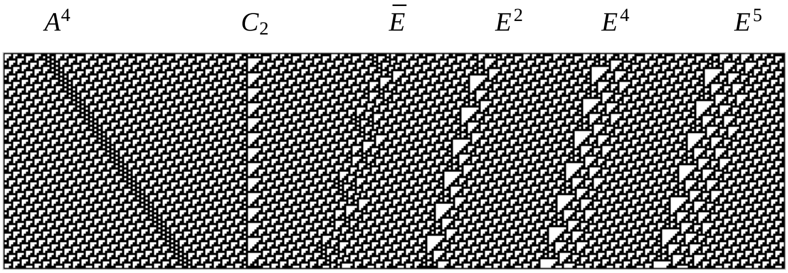


FIGURE 1
Space-time configuration of propagating patterns A^4, $\bar{E}$, E^2, E^4, and E^5 and stationary pattern C_2 necessary to realize computation on the array of rule 110. The time goes from the top to the bottom.

appendant to the end of the tape according to a fixed appendant table, while an '0' causes the appendant to be skipped. An appendant is cyclically chosen from the appendant table. In spite of these restrictions, CTSs are equivalent to tag systems. The components necessary to perform computation by CTS are composed of the stationary or propagating patterns shown in Figure 1. A detailed explanation of the emulation of CTS by rule 110 is found in Ref. [7].

The computation process of CTS we deal with in this article is implemented on the array of rule 110 as schematically shown in Figure 2. The time goes from the top to the bottom. CTS emulation process is primarily performed by four components, tape data, ossifier, leader, and appendant. The array size is 65,900 although only the middle part of the whole array with about 20,000 cells is shown in Figure 2 and periodic boundary conditions are employed. The emulation process reaches stationary configuration at about $t = 54,500$ (time starts from $t = 0$). The initial configuration file that can create this process can be downloaded at the web site [8].

3 POWER SPECTRA OF CYCLIC TAG SYSTEM EMULATION

Let $s_x(t) \in \{0, 1\}$ denote the value of site x at time step t in a CA. The discrete Fourier transform of a time series of states $s_x(t)$ of the site x for $t = 0, 1, ..., T-1$ is given by

$$\hat{s}_x(f) = \frac{1}{T}\sum_{t=0}^{T-1} s_x(t)\exp(-i\frac{2\pi t f}{T}), \qquad f = 0, 1, \cdots, T-1. \tag{1}$$

We define the power spectrum of CA as

$$S(f) = \frac{1}{N}\sum_{x=0}^{N-1} |\hat{s}_x(f)|^2, \tag{2}$$

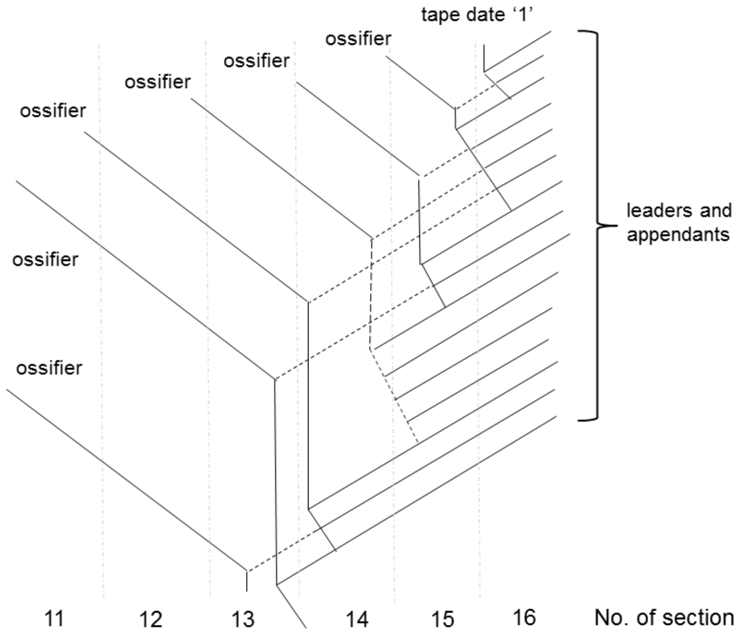


FIGURE 2
Diagram of the emulation process of cyclic tag system by rule 110. The time goes from the top to the bottom.

where N denotes the total number of sites and the summation is taken in all sites. The period of the component at a frequency f in a power spectrum is given by T/f. The least square fitting of power spectrum $S(f)$ by

$$\ln(S) = \alpha + \beta \ln(f), \tag{3}$$

from $f = 1$ to $f = f_b$ gives the exponent β. The residual sum of squares σ^2 is given by

$$\sigma^2 = \frac{1}{f_b} \sum_{f=1}^{f_b} (\ln(S) - \alpha - \beta \ln(f))^2. \tag{4}$$

Since the behaviour during the computation process varies considerably according to the location on the array, we divide the whole array into 20 sections (section $0 \sim 19$ starting from the left) consisting of 3, 295 cells and

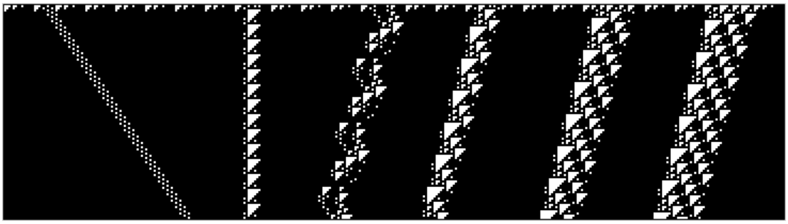


FIGURE 3
Filtered space-time pattern of Figure 1.

calculate the power spectrum of each section individually. We set $N = 3,295$ in Equation (2). The way of dividing the whole array is after the example of Ref. [9] in which the computation process of rule 110 is analyzed by means of LZ complexity. The locations of section $11 \sim 16$ are depicted in Figure 2. As the computation process ends at about $t = 54,500$, we set $T = 55,000$ in Equation (1). Since we are interested in the long term correlation during the computation process, we use $f_b = 100$ in Equation (4) to focus on the power at low frequencies.

One of the most remarkable feature of the evolution of rule 110 is the periodic background with period seven. It is pointed out that $1/f$-type characteristics of the evolution starting from random configuration can be made clear by removing the periodic background [10]. We call a space-time pattern obtained by removing the periodic background from the original one 'filtered' space-time pattern. Figure 3 is the filtered space-time pattern obtained from the one shown in Figure 1. In this research we use the filtered space-time pattern to calculate power spectra besides the original one to elucidate the essential feature of the dynamics of computation process in rule 110. The power spectra of each section are shown in Figure 4. The original power spectra are located on the left column and the filtered ones are on the right. Both the x and y axes are plotted on a logarithmic scale.

Figure 5 shows the exponent β (left) and the residual sum of squares σ^2 (right) of each section estimated by Equations (3), (4) in the range of frequencies $f = 1 \sim 100$. In section $13 \sim 16$, the exponents are in the range between -1 ± 0.2 both in the original and filtered power spectra. So we can conclude that the behaviour exhibits $1/f$ noise in the most actively interacting area among the whole array as shown in Figure 2.

The residual sums of squares σ^2 in section $13 \sim 16$ have smaller value in the filtered power spectra than in the original ones. The smaller the residual sum of squares σ^2 is, the more closely the power spectrum fits into the power law. In other words, by removing periodic background from the space-time pattern, the evolution gets close to $1/f$-type behaviour. This result implies

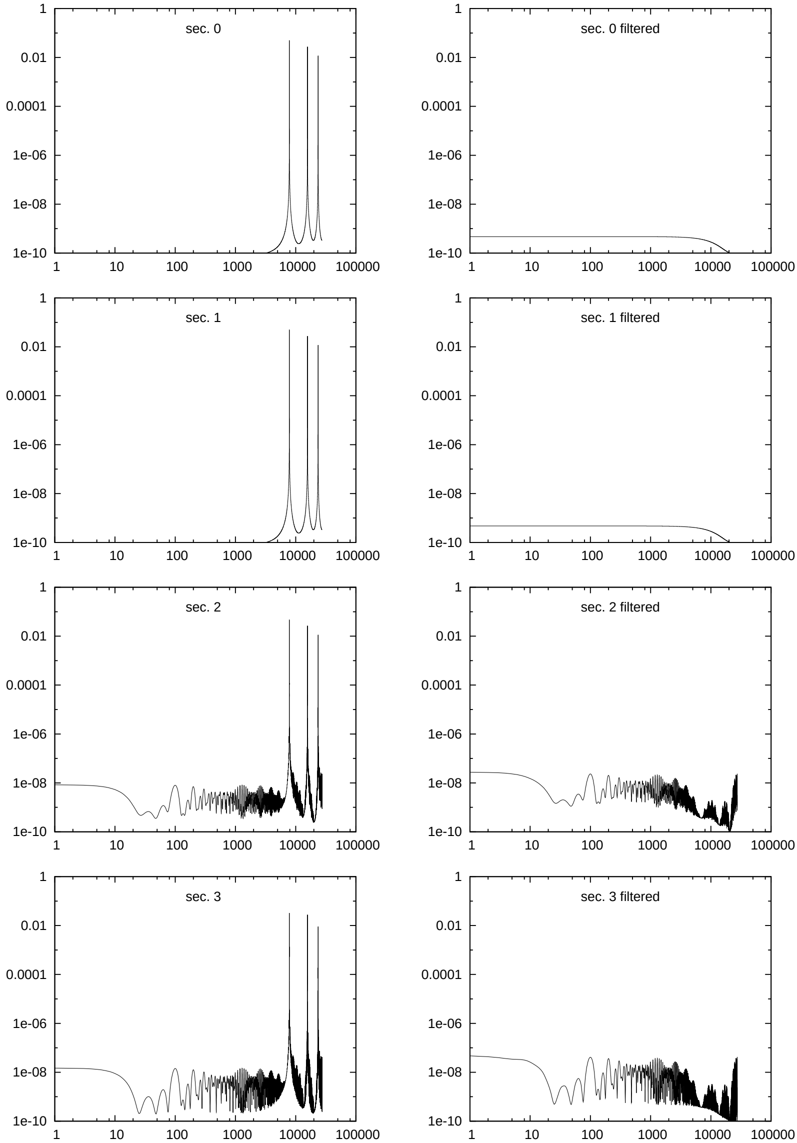
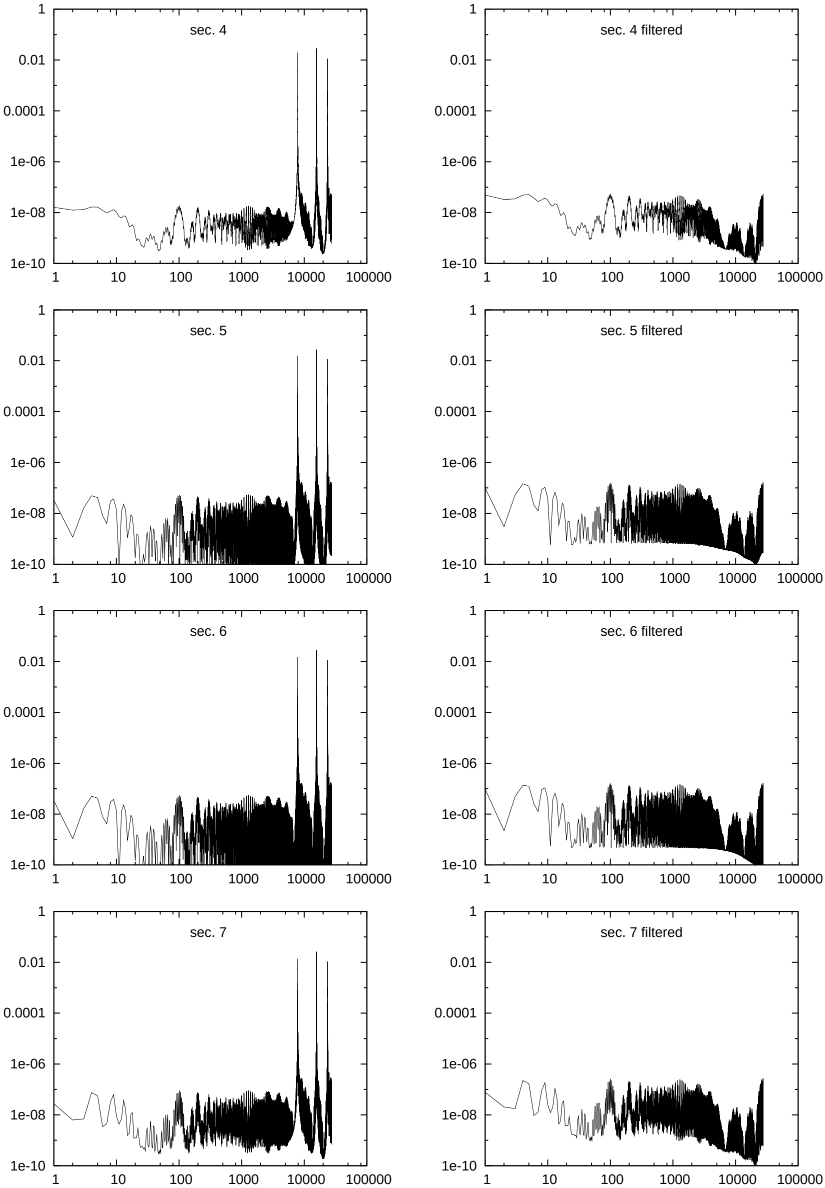


FIGURE 4
Power spectra of the original evolution (left) and the filtered one (right) in each section. The x-axis is the frequency f, y-axis is power S. Both axes are plotted on a logarithmic scale.



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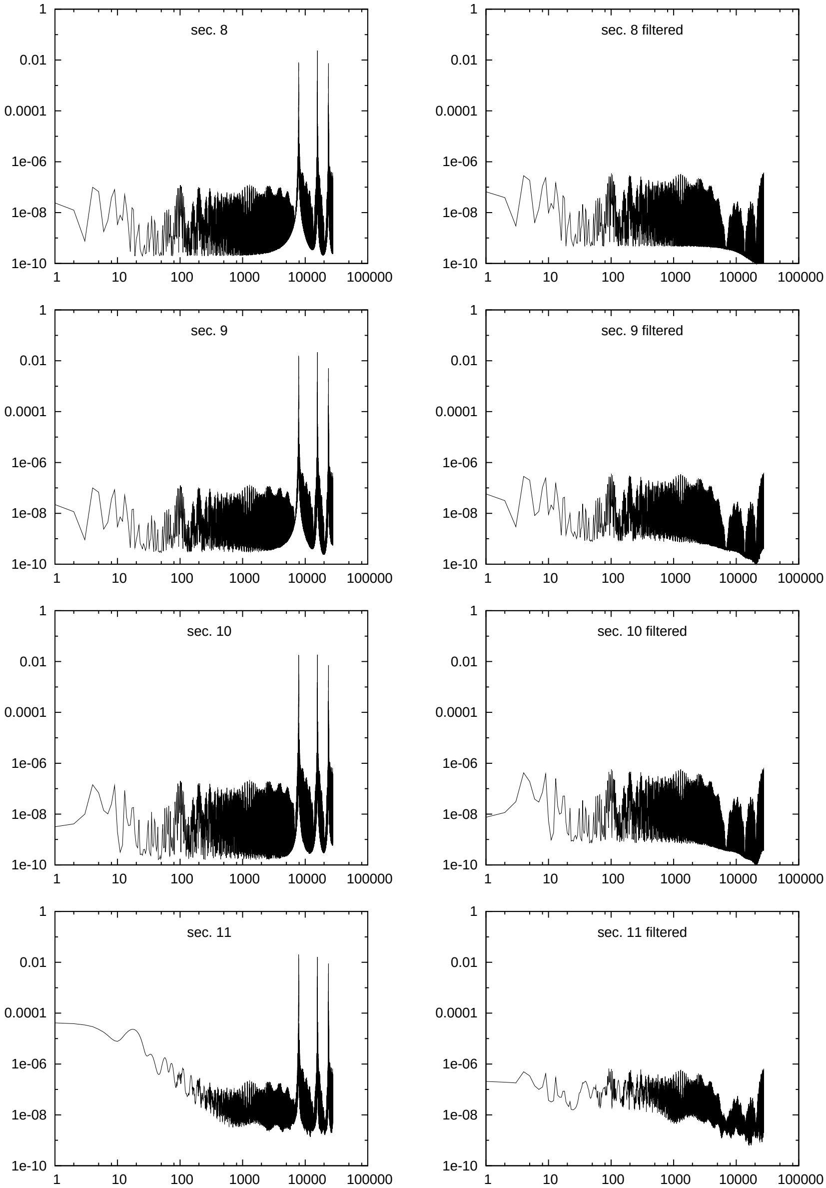

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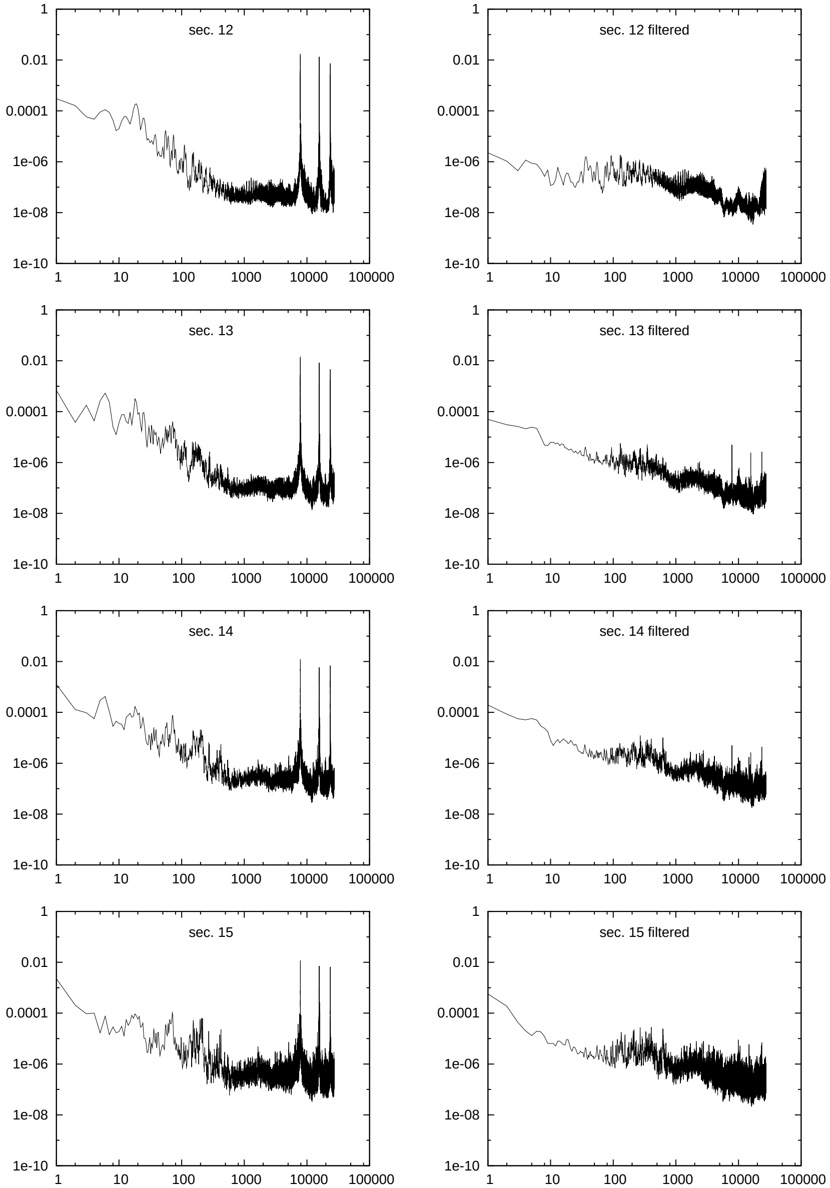

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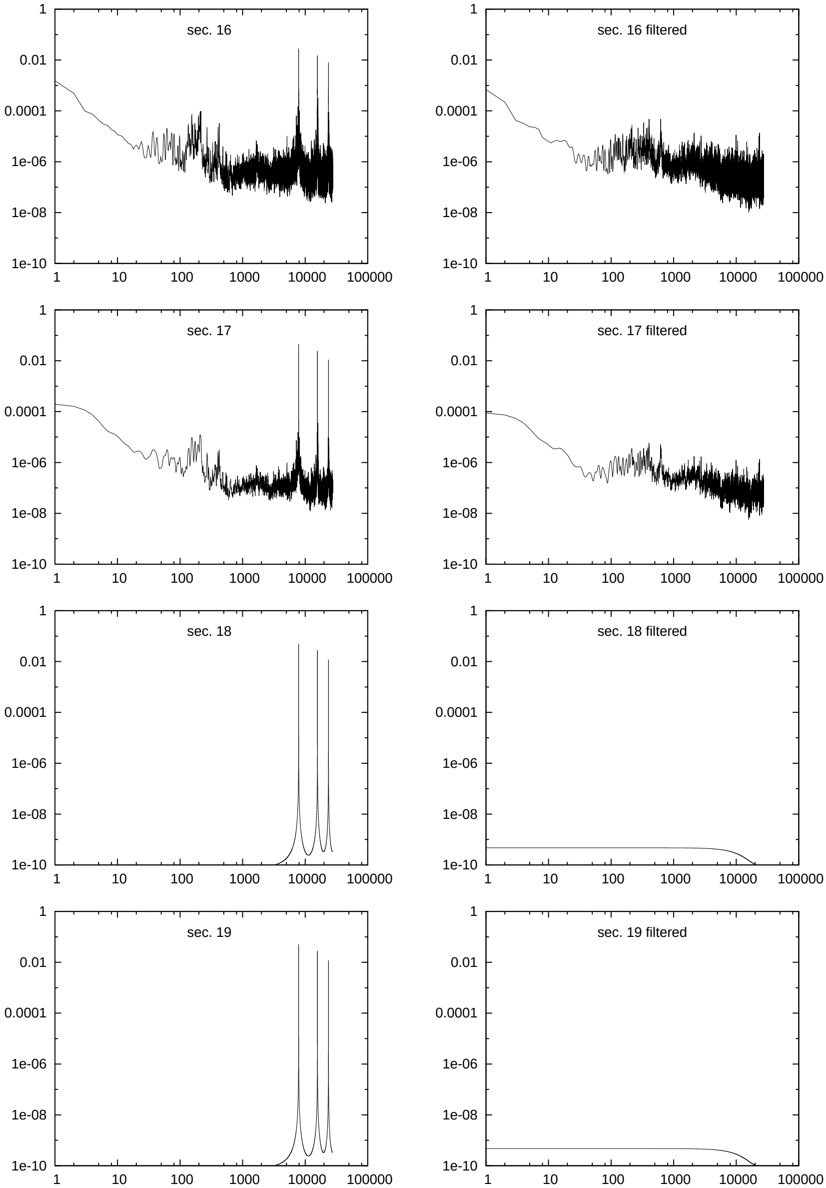

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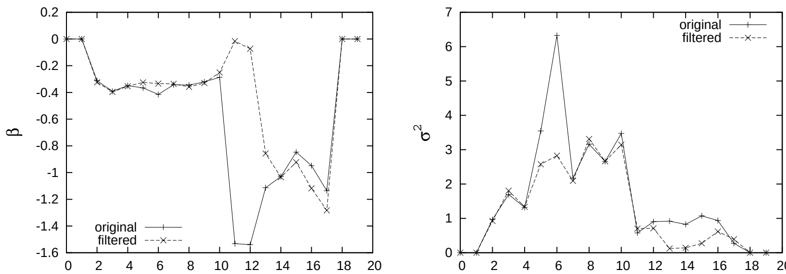

FIGURE 5
Exponent β (left) and the residual sum of squares σ^2 (right) of the of power spectrum of each section calculated in the range of frequencies between $f = 1 \sim 100$.

that the origin of $1/f$-type power spectra in the evolution of computation process in rule 110 is contained in the stationary and propagating patterns and their interaction. The exponent β of section 17 is in the range between -1 ± 0.2 only in the original power spectrum, not in the filtered one. This is because the power at low frequencies becomes flat as shown in Figure 4. This kind of power spectrum is called Lorentzian spectrum that is caused by a fluctuation with finite time constant. As the gliders are wiped off the array at about $t = 9,900$ in section 17, that seems to bring a finite relaxation time into the evolution. The same thing can be said of section 11.

In section 0, 1, 18, and 19 where there is no patterns except for periodic background, the power spectrum of the original evolution is characterized solely by sharp peaks caused by the periodic background. In section $2 \sim 10$ there are only a few sparsely scattered shifting patterns such as ossifier, appendant or leader without no collisions. That causes white noise in the filtered power spectra.

In section 11 and 12 there is a considerable distinction between the original and filtered power spectra. The exponents β of the original power spectra in both sections are about -1.5, while those of the filtered ones are close to zero. The exponent β of the original power spectrum in section 12 is -1.539 that means the behaviour in this section has something in common rather with Brownian noise than with $1/f$ noise. The Brownian motion has the power spectrum with the exponent $\beta \approx -2$ and hence Brownian noise is named after it. The original power spectrum of section 11 is considered to be Lorentzian as its power at low frequencies becomes flat.

Figure 6 shows the power spectra averaged over all cells in the array calculated from the original evolution (left) and the filtered one (right). As the exponent β of the power spectrum is -1.098 in the original evolution and -0.982 in the filtered one, both of them are considered to be $1/f$ noise.

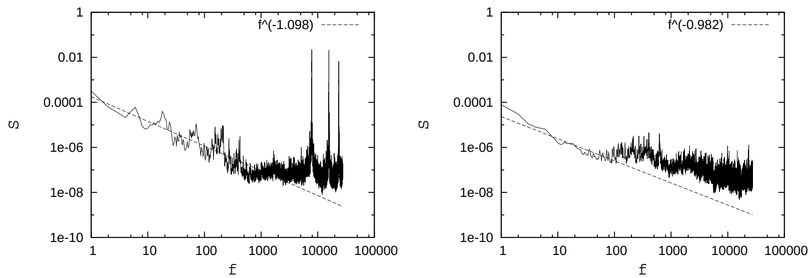

FIGURE 6
Power spectra of the computation process averaged over all cells in the array. Left: original evolution ($\beta = -1.098$, $\sigma^2 = 0.602$), Right: filtered one ($\beta = -0.982$, $\sigma^2 = 0.213$). β and σ^2 are estimated in the range of $f = 1 \sim 100$.

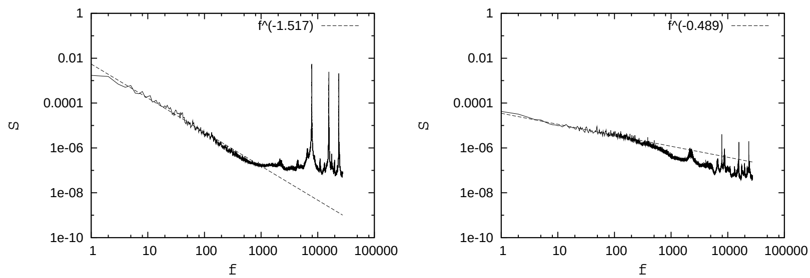

FIGURE 7
Power spectra of the evolution starting from a random configuration with array size 65,900 for $T = 55,000$. Left: original evolution ($\beta = -1.517$, $\sigma^2 = 0.076$), Right: filtered one ($\beta = -0.489$, $\sigma^2 = 0.028$). β and σ^2 are estimated in the range of $f = 1 \sim 100$.

On the other hand, Figure 7 shows the power spectra calculated under the same conditions except for starting from a random configuration. The power spectrum calculated from the original evolution becomes flat at low frequencies that implies the evolution reaches stationary configuration within the observed time steps, while β is -0.489 in the filtered evolution and it is not considered to be $1/f$ noise. In both cases, in original and filtered power spectrum, the evolution starting from a random configuration does not exhibit $1/f$ characteristics in these settings.

4 DISCUSSION

We calculated the power spectra of the CTS emulation process by rule 110 in the original and filtered evolution. The power spectra exhibit various types varying from section to section such as Lorentzian, white noise, Brownian noise and $1/f$ noise that is observed in the most actively interacting sections. The power spectrum averaged over the whole array shows $1/f$ noise.

Intermittency is considered to be one of the main mechanisms to produce $1/f$ noise [11] in which periodic behaviour is disrupted occasionally and irregularly by chaotic behaviour called “burst”. From the dynamical point of view, intermittency might be occurring in the most actively interacting sections such as section $13 \sim 16$ where periodic phases created by propagating patterns shifting without collision are disrupted by bursts caused by the collision with stationary patterns. From the viewpoint of computation, the computation process needs three kinds of functions on information, that is, transmission, storage, and operation of information. The transmission of information in rule 110 is achieved by gliders and the storage is accomplished by stationary patterns, while the operation of information is carried on by the interaction between these patterns.

There might be a possibility that these two kinds of dynamics, one accompanied with intermittency and the other capable of performing computation, overlap each other in CA rule space. This guess reminds us of the hypothesis of “the edge of chaos” [12, 13] that says there arises a kind of phase transition from regular to chaotic behaviour when traversing a CA rule space and CAs with the ability to perform universal computation are located at the phase transition. It is known that the average transient length from a random initial configuration grows rapidly in the vicinity of the edge of chaos; this is known as the critical slowing down in the study of phase transitions. On the other hand, $1/f$ noise in rule 110, especially power law at low frequencies, is generated by transient behaviour during the evolution. For example the exponent β gets close to zero at low frequencies in case of the evolution with transient length shorter than the observation length T. Therefore $1/f$ noise in CAs is accompanies by long transient in the evolution. Putting all accounts together, the presence of $1/f$ noise might be able to be a measure to detect the edge of chaos.

The power spectra (Figure 6) averaged on the whole array have $1/f$ characteristics more obviously in the computation process than in the evolution starting from random configuration (Figure 7). We can guess that the cooperation between the dynamics of rule 110 that potentially supports universal computation and the initial configuration that is explicitly prepared to develop a computation process prominently brings long-running intermittency and $1/f$ noise. On the other hand, the evolution starting from random configuration can not produce transient behaviour long enough to generate $1/f$ noise properly because its initial configuration sadly lacks a kind of “complexity” that is supposed to be contained in the initial configuration of CTS emulation.

Here arises a question how power spectrum correlates with the encoding method of computation process in rule 110. For example, let us imagine that we construct an erroneous configuration in which a slight defect is contained such as the wrong distance between a leader and an appendant or an inappropriate phase of a leader at the time of collision with tape data. It is obvious that the trivial mistake in initial configuration does not lead the evolution into correct computation process. However, we can not say for certain whether the evolution starting from erroneous configuration exhibits intermittency enough to generate $1/f$ noise. There is room for further investigation on this point.

There is a broad distinction in the exponent β between the original and filtered power spectra in section 11 and 12. The original evolution in section 12 exhibits Brownian noise while the filtered one exhibits white noise. The section 12 distinctively has a lot of ossifiers monotonically shifting from the left to the right. On the contrary the original evolution in section 11 exhibits Lorentzian power spectrum. Taking the fact that section 11 has a fewer number of ossifiers than section 12 into consideration, we can guess that the density of ossifier is crucial to the exponent of power spectrum. At present, however, we can not give an acceptable explanation of the appreciable difference in power spectrum between the original and filtered evolution in section 11 and 12.

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REFERENCES

[1] Cook, M., (2004). Universality in elementary cellular automata, *Complex Systems*, 15, 1–40.

[2] Ninagawa, S., (2008). Power spectral analysis of elementary cellular automata. *Complex Systems*, 17, 399–411.

[3] Keshner, M.S., (1982). $1/f$ noise. *Proc. IEEE,* 70, 211–218.

[4] Berlekamp, E.R., Conway, J.H. and Guy, R.K., (1982). Winning Ways for Your Mathematical Plays Vol. 2. New York: Academic Press.

[5] Ninagawa, S., Yoneda, M. and Hirose, S., (1998). 1/f fluctuation in the "Game of Life" *Physica D,* 118, 49–52.

[6] Post, E., (1943). Formal reductions of the combinatorial decision problem, *American Journal of Mathematics,* 65 (2), 197-215.

[7] Martínez, G.J., McIntosh, H.V., Seck-Tuoh-Mora, J.C., Vergara, S.V.C., (2011). Reproducing the cyclic tag system developed by Mattew Cook with rule 110 using the phase f_{i}_1. *J. Cellular Automata*, 6, 121–161.

[8] Martínez, G.J. Elementary cellular automaton rule 110 http://uncomp.uwe.ac.uk/genaro/rule110/ctsRule110.html

[9] Ninagawa, S. and Martínez, G.J., (2014). Compression-based analysis of cyclic tag system emulated by rule 110. *J. Cellular Automata*, 9, 23–35.

[10] Ninagawa, S., (2015) Dynamics of universal computation and 1/f noise in elementary cellular automata. *Chaos, Solitons and Fractals*, 70, 42–48.

[11] Manneville, P., (1980). Intermittency, self-similarity and $1/f$ spectrum in dissipative dynamical systems. *J. Physique*, 41, 1235–1243.

[12] Langton, C., (1990). Computation at the edge of chaos: phase transitions and emergent computation. *Physica D*, 42, 12–37.

[13] Li, W., Packard, N.H. and Langton, C.G., (1990) Transition phenomena in cellular automata rule space. *Physica D*, 45, 77–94.

[14] Ninagawa, S. and Martínez, G.J., (2014). Power spectral analysis of the computation process by rule 110. In *Cellular Automata: 11th International Conference on Cellular Automata for Research and Industry, ACRI 2014, Krakow, Poland, September 2014, Proceedings*. pages 22-25. Springer.