Some Notes About the Game of Life Cellular Automaton

Genaro J. Martínez, Andrew Adamatzky, and Juan C. Seck-Tuoh-Mora

Abstract This is a short review of selected results related to John Conway's Game of Life cellular automaton. The review is based on our participation in the "A Tribute to Conway: A Lectures Series on the Memory of John Horton Conway" (https://youtu.be/WqKkmfOt9Ww), celebrated virtually in India and organized by Sukanta Das and Kamalika Bhattacharjee in 2020. Additional contributions are made by Andrew Adamatzky and Juan C. Seck-Tuoh-Mora.

Keywords John H. Conway · Game of Life · Life-like rules · Cellular automata · Gliders

1 The Game of Life

The Game of Life is an elegant, simple and compact semi-totalistic function that brings together artificial life, complex system, emergent behavior and non-linear systems. The Game of Life cellular automaton is the most famous rule into the cellular automata literature and one of the most researched during most of 50 years. There are two excellent repositories where you can explore any Life objects and recently discovered Life patterns and complex structures: *Conway's Game of Life* http://www.conwaylife.com/ and *LifeWiki* https://www.conwaylife.com/wiki/.

G. J. Martínez (✉)
Artificial Life Robotics Lab, Escuela Superior de Cómputo, Instituto Politécnico Nacional, Mexico City, Mexico
e-mail: gjuarezm@ipn.mx; genaro.martinez@uwe.ac.uk

G. J. Martínez · A. Adamatzky
Unconventional Computing Lab, University of the West of England, Bristol, United Kingdom
e-mail: andrew.adamatzky@uwe.ac.uk

J. C. Seck-Tuoh-Mora
Area Académica de Ingeniería y Arquitectura, ICBI, Universidad Autónoma del Estado de Hidalgo, Hidalgo, Mexico
e-mail: jseck@uaeh.edu.mx

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Between 1969 and 1970 Conway designed and published his famous two-dimensional cellular automaton "The Game of Life" in the popular column of *Scientific American* edited by Martin Garden [14].

The Game of Life is recognized as the second important stage in the cellular automata literature after John von Neumann self-assembling machines (see [23]). Main difference between von Neumann and Conway automata is the kind of function, while von Neumann used an orthogonal neighbourhood, Conways used an isotropic relation in two dimensions, it is the Moore neighbourhood. The Game of Life uses a binary alphabet $\Sigma = \{0, 1\}$, where state one depicts alive organisms and state zero nothingness (empty space).

The Game of Life is the evolution (semi-totalistic) rule $R(2333)$ (Carter Bays notation) or $B3/S23$. The rule belongs satisfies the following conditions.

Birth: an empty cell adjacent to exactly 3 neighbours is a birth cell the next time.

Survival: a live cell with 2 or 3 neighbouring counters survives for the next generation.

Death: a cell with 4 or more neighbours dies (becomes empty) from overpopulation. Every live cell counter with 1 neighbour or none dies (becomes empty) from isolation.

Conway proposed two characteristics that were key to induce non-trivial behaviour in the Game of Life.

- The function will not disappear quickly (equilibrium).
- The function will grow forever (expansion).

This combination would produce a number of active cells with possibilities of interacting in different landscapes. So, several patterns were proposed and the *acorn*[1] pattern was one of the most interesting configurations, found in 1971. A role of the acorn pattern is historically important in the search of universe of small configurations with unpredictable evolution.

Acorn is an excellent example where a simple (compact) pattern can evolve to patterns with a non-trivial behaviour in long span of time. Figure 1a illustrates the acorn pattern shaped by seven cells in state one in the lattice of 3×7 cells. In Fig. 1b we can see the density history during its evolution which grew quasi-constantly before reaching its stability. So, Fig. 1c presents a three-dimensional projection of this acorn evolution concatenating every two-dimensional plane successively, where chaotic regions do not stop or emerge and some few gliders escape from the central area. Acorn evolution reached its stability in generation 5206 with a final population of 633 cells.

Particularly, David Eppstein specifies that Wolfram's classes can be related as patterns [11].

[1] https://www.conwaylife.com/wiki/Acorn.

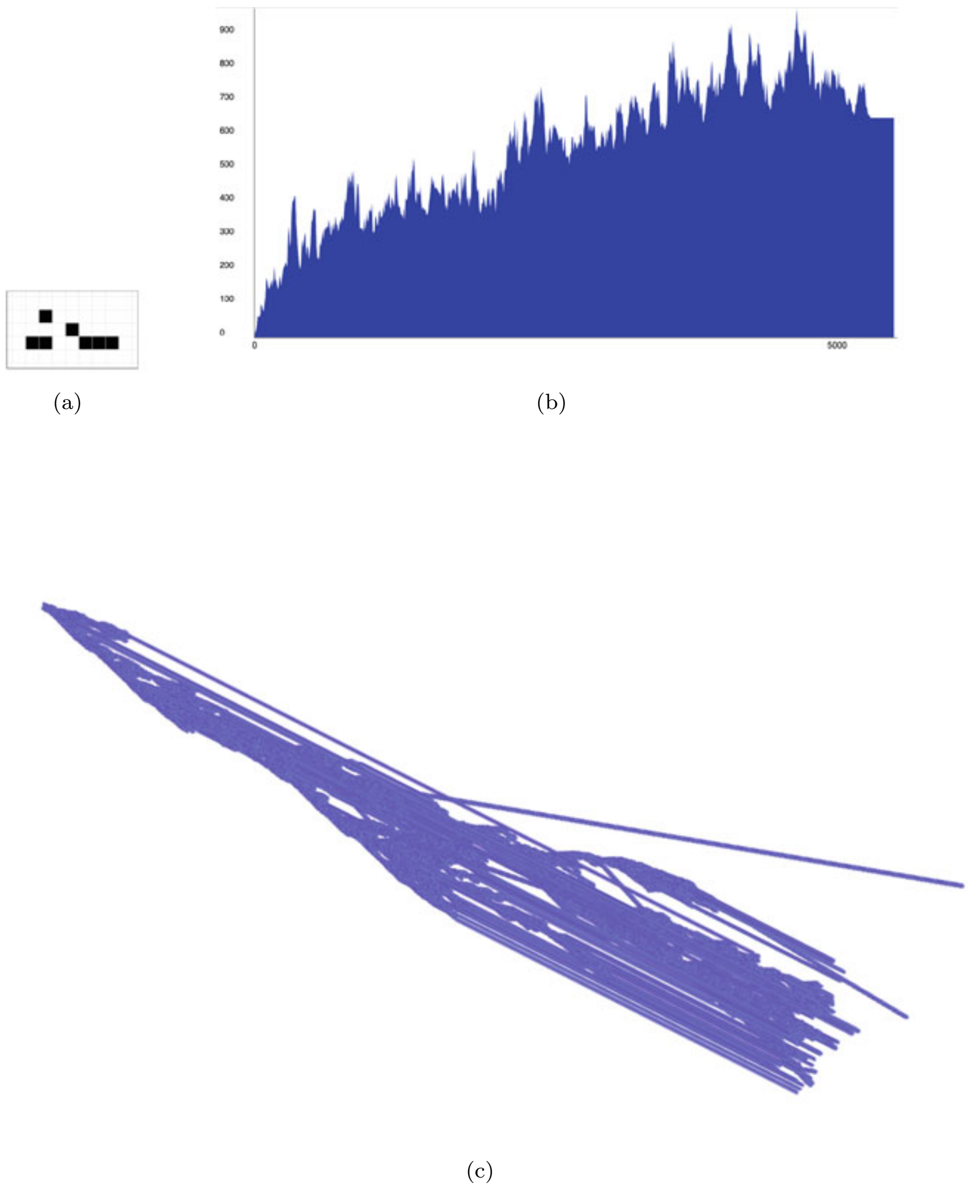


Fig. 1 **a** Acorn pattern shaped by seven cells in state one. **b** History density evolution of acorn. **c** Three-dimensional projection of acorn evolution (a video of this evolution is available from https://youtu.be/NADVWj1-KS4)

1. *Evolution leads to a homogeneous state.*
2. *Evolution leads to a set of separated simple stable or periodic structures.*
3. *Evolution leads to a chaotic pattern.*
4. *Evolution leads to complex localized structures, sometimes long-lived.*

In this direction, the Game of Life belongs to class 4 where well-defined mobile, periodic or stable localized structures emerge during evolution. A way to try to understand this characterization is with the mean field approximation proposed by

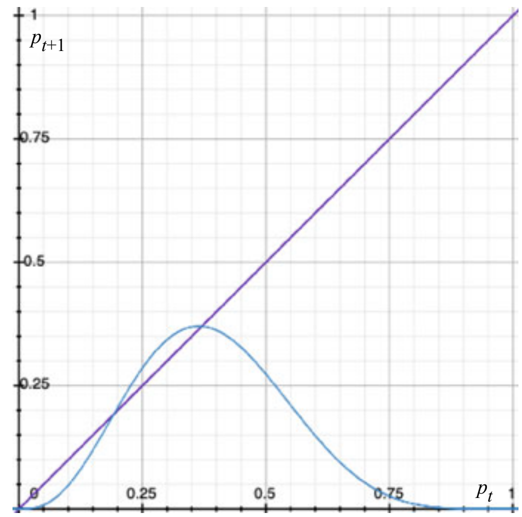


Fig. 2 Mean field curve for the Game of Life

Howard Gutowitz [17]. Mean field approximation is a useful tool which calculates averages of intervals assigning probabilities to each element of the alphabet expressed as a polynomial. This approximation assumes that the elements are independent. This way, across the number of fixed points Harold V. McIntosh characterized this classification as follows [22]:

1. *Monotonic, entirely on one side of the diagonal.*
2. *Horizontal tangency, curve never reaches diagonal.*
3. *No tangencies, curves cross diagonal.*
4. *Horizontal plus diagonal tangency, no crossing.*

The Game of Life polynomial is the following: $p_{t+1} = 84\, p_t^3\, q_t^6 + 56\, p_t^4\, q_t^5$. So, its graphical curve is illustrated in Fig. 2. The first stable fixed point at the origin guarantees its stable state $p_{t+1} = 0$, the second unstable point $p_{t+1} = 0.1986$ relates to areas of densities where the space–time dynamic is unknown. The last stable point in $p_{t+1} = 0.37$ indicates that the Game of Life will converge almost surely to configurations with small densities of states one.

Some relevant results in the Game of Life are enumerated below. The history of relevant results in the Game of Life is filled with accumulative results from a large number of researchers, which begins with the famous newsletter *Lifeline* [27] which reincarnated in a very-well organized and specialized site *Forums for Conway's Game of Life*.[2]

1. Register machine (Conway, 1982) [6].
2. Turing machine (Paul Rendell, 2001) [26].
3. Life universal computer (Paul Chapman, 2002) [7].
4. Algorithms to find complex patterns (David Eppstein, 2002) [12].
5. Still life theory (Matthew Cook, 2003) [10].

[2] https://www.conwaylife.com/forums/.

6. Spartan universal computer-constructor (Adam P. Goucher, 2009) [16].

Newsletters and books dedicated to the Game of Life (chronological order).

1. Lifeline newsletter (Robert Wainwright, 1971) [27].
2. The Recursive Universe (William Poundstone, 1985) [25].
3. New Constructions in Cellular Automata (David Griffeath, Cris Moore (Eds.), 2003) [15].
4. Game of Life Automata (Andrew Adamatzky (Ed.), 2010) [1].
5. Universal Turing machine (Paul Rendell, 2016) [26].

Some particular patterns (chronological order).

1. Glider (Richard K. Guy, 1969) https://www.conwaylife.com/wiki/Glider.
2. Glider gun (Bill Gosper, 1970) https://www.conwaylife.com/wiki/Gosper_glider_gun.
3. Puffer train (Gosper, 1971) https://www.conwaylife.com/wiki/Puffer_1.
4. Eater (Gosper, 1971) https://www.conwaylife.com/wiki/Eater_1.
5. Garden of Eden (Roger Banks, 1971) https://www.conwaylife.com/wiki/Garden_of_Eden.
6. Self-replicator (Dave Greene, 2013) https://www.conwaylife.com/wiki/Linear_propagator.

Some semi-totalistic functions, variants, and projections (chronological order).

1. Inkspot, renamed as Life without Dead (Tommaso Toffoli, Norman Margolus, 1987) https://www.conwaylife.com/wiki/OCA:Life_without_death.
2. Three dimensions (Bays, 1987) [4].
3. HighLife (Nathan Thompson, 1994) https://www.conwaylife.com/wiki/OCA:HighLife.
4. Triangular, Pentagonal, Hexagonal (Bays, 1994) https://cse.sc.edu/~bays/CAhomePage.
5. Hexagonal (Paul Callahan, 1997) http://www.radicaleye.com/lifepage/hexrule.txt.
6. Seeds (Brian Silverman, 1996) https://www.conwaylife.com/wiki/OCA:Seeds.
7. Larger-than-Life (Kellie Michele Evans, 1996) https://www.conwaylife.com/wiki/Larger_than_Life.
8. Penrose (M. Hill, S. Stepney, F. Wan, 2005) https://www-users.cs.york.ac.uk/susan/bib/ss/nonstd/penroselife.htm.
9. Four dimensions (Bays, 2009) [5].

Some systematic characterizations in the Life-like rules were reported initially by Magnier et al. in 1997 [24], some years later Adamatzky et al. in 2010 explore the full range of semi-totalistic rules [2].

In [2] we represent dynamical complements as illustrated in Fig. 3 morphology-based classification. Stable orbit matches uniform behavior with nill density of cells

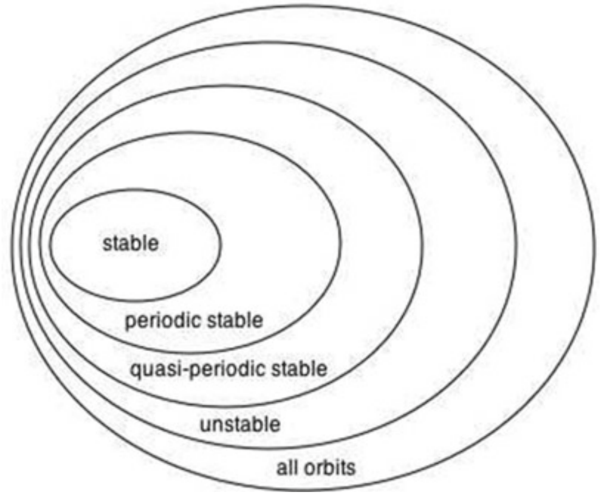


Fig. 3 Diagram of dynamical complements of morphological classification

in state 1. The periodic orbit is typical for configurations that are usually dominated by stationary localizations, still life and cycle life. Quasi-stable density is a class where cellular space is dominated by quasi-periodic density regions, they have very close density values although they are not exactly in the same position. It is known as collective behaviour [8]. Unstructured and unstable density represents chaotic behaviour. The last class is characterized by "indefinite" density and complex behavior (Fig. 3).

The Game of Life is a robust complex rule. A cellular automaton has robust dynamics with respect to a composition function if such dynamics preserve emergent behaviour later of such composition (for details see [18]). We compose the Game of Life function with a function of memory. Particularly, we use the majority and minority memory functions (Fig. 4).

Cellular automata with memory are an extension of the original model in such a way that every cell x_i is allowed to remember its states during some fixed period of its evolution. Cellular automata with memory have been proposed originally by Alonso-Sanz [3]. This way, we implement a memory function ϕ, as follows: $s_i^{(t)} = \phi(x_i^{t-\tau+1}, \ldots, x_i^{t-1}, x_i^t)$, where $1 \leq \tau \leq t$ determines the *degree of memory*. Thus, $\tau = 1$ means no memory (or conventional evolution), whereas $\tau = t$ means unlimited trailing memory. Each cell's trait $s_i \in \Sigma$ is a state function of the series of states of the cell i with memory backward up to a specific value τ. In the memory implementations run here, commences to act as soon as t reaches the τ time-step. Initially, i.e., $t < \tau$, the automaton evolves in the conventional way. Later the original rule is applied on the cell states s as: $\varphi(\ldots, s_{i-1}^{(t)}, s_i^{(t)}, s_{i+1}^{(t)}, \ldots) \rightarrow x_i^{t+1}$ to get an evolution with memory. Thus in cellular automata with memory, while the mapping φ remains unaltered, historic memory of all past iterations is retained by featuring each cell as a summary of its past states from ϕ. We can say that cells canalises memory to the map φ [3].

Other research done by Nazim Fatés shown that the Game of Life is robust from asynchronous version, for details see [13].

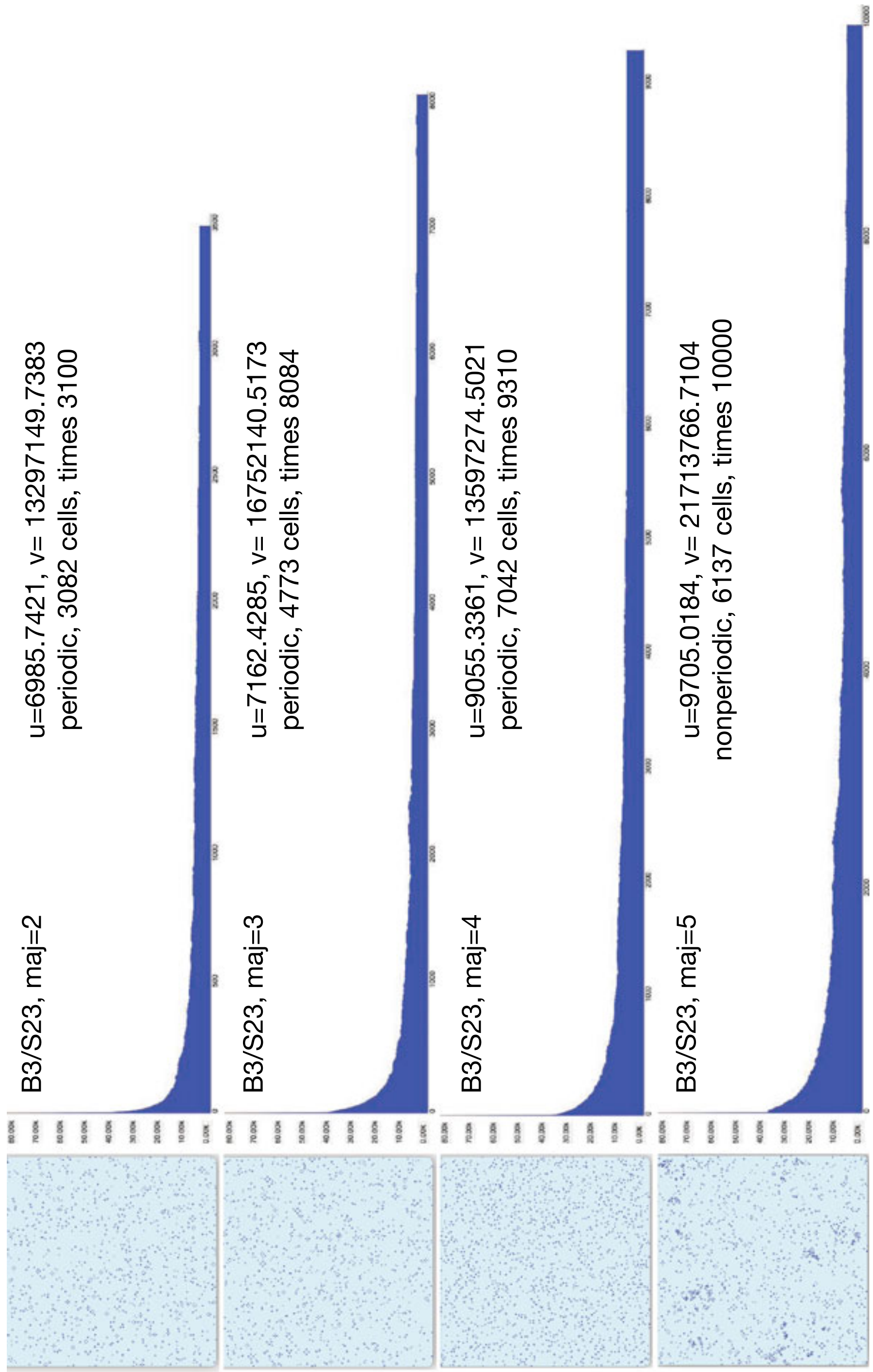


Fig. 4 The Game of Life is a robust complex rule. The Game of Life is composed with a majority memory function, starting with a random initial condition to 50% and running to 10,000 steps (snapshots) in an evolution space of 400×400 cells. The memory function uses a range from $\tau =$ 2 to 5. The density history is followed to reach a periodic state or not

2 The Game of Life and Its Connection in One Dimension

Rule 22 is the natural projection to one dimension and initially studied by McIntosh in 1990 [23]. Rule 110 is proposed as LeftLife by Cook in 1999 [9].

Elementary cellular automaton rule 22 is one dimensional projection of the Game of Life automaton(for details see [23]). Although the Rule 22's global behaviour does not show outstandingly complex dynamics, rule 22 is classified as a chaotic rule (class 3) in the Wolfram's classification [28].

Rule 22 can be seen as a natural projection to the Game of Life [6] given in the next conditions [23]:

Birth: a dead cell x_i at the time t will be born in $t+1$ if there is just one live neighbour.

Survival: an alive cell x_i at the time t will survive in $t+1$ if there are no live neighbours

Death: an alive cell x_i at the time t will be dead in $t+1$ if there are just two or one live neighbours, it is dead by overcrowding.

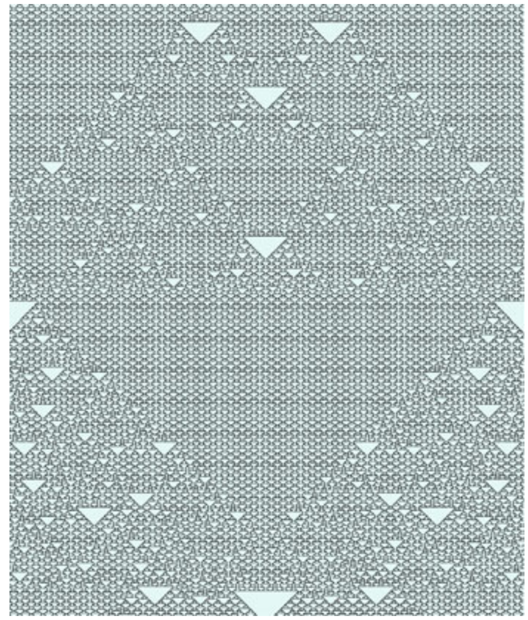
Such a relation covers conditions of the Game of Life. Nevertheless, from a quick exploration in the one-dimensional dynamics, it does not exhibit complex behaviour. Rule 22 is an elementary cellular automaton evolving in one dimension of order $|\Sigma| = 2$ and neighbourhood radius $r = 1$. Thus the local rule φ is defined as follows:

$$\varphi_{R22} = \begin{cases} 1 \text{ if } 100, 010, 001 \\ 0 \text{ if } 111, 110, 101, 011, 000 \end{cases} . \tag{1}$$

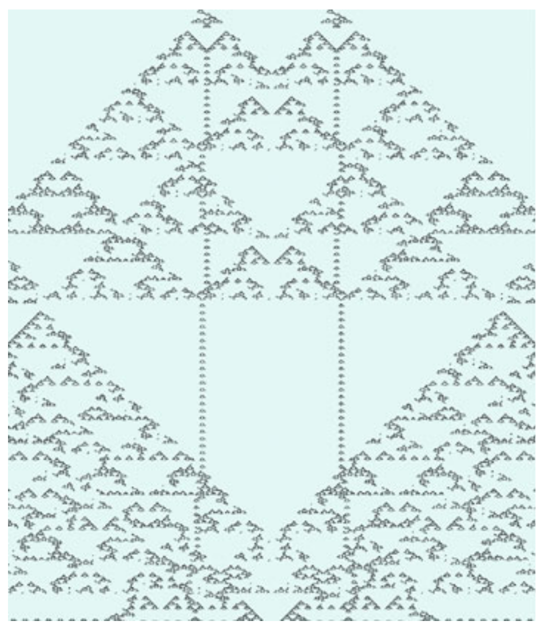
The local function φ_{R22} has a probability of 37.5% to get states 1 in the next generation and consequently a higher probability to get state 0 in the next generation. Of course, it is the same fixed point value for the Game of Life.

Recently in [21] it was demonstrated that rule 22 is able to support complex behaviour, including non-trivial travelling patterns, as gliders. Figure 5 illustrates the collisions between two fractals propagating in one dimension. The probability to get this initial configuration is very slow because typical evolution is chaotic in this rule. To reproduce the collisions between fractals emerging in rule 22 we need the symbolic equation: $e^* - 11 - e^{11} - 11 - e^*$, where the symbol '–' means a concatenation operation, the periodic background (or ether) is determined by the string 11101110111011100000. Figure 5a shows the original evolution on a ring of 1,164 cells where both fractals start at the center of the window and evolve during 1,049 generations. Here it is possible to distinguish that the composition of both fractals preserves its structure yielding a reaction between multiple fractals. Another kind of fractals and collisions can be explored in [21]. Figure 5b shows the same evolution but filtered, this technique permits to separate the mosaic with most frequency in the evolution space and the patterns are more clear to see. Also, we can see that these fractals evolve with two stationary particles that travel with small displacements produced by perturbations when they collide with the fractal structures.

Fig. 5 A non-typical evolution of elementary cellular automaton rule 22, two fractals evolve and collide on a periodic background. **a** This initial condition is determined by the regular expression $e^* - 11 - e^{11} - 11 - e^*$ in a ring of 1,164 cells evolving in 1,049 steps. **b** The evolution is the same a in (**a**) but a filter is selected to visualise non-trivial complex patterns emerging in this automaton [21]



(a)



(b)

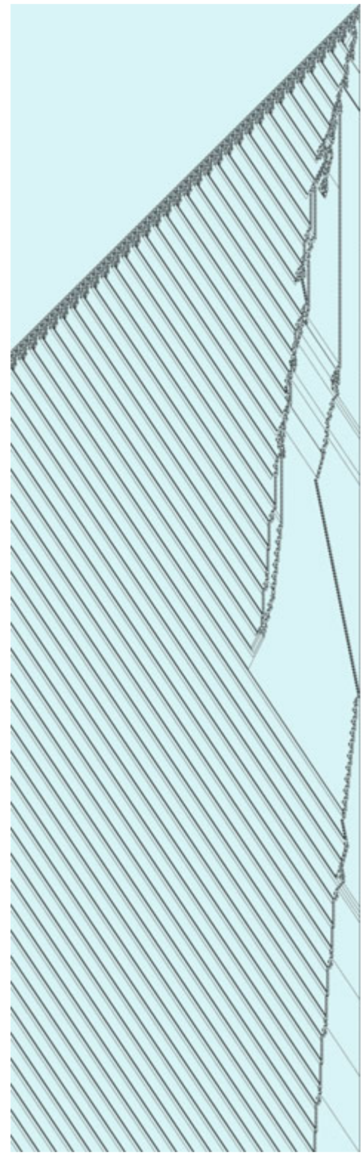


Fig. 6 Evolution of a pentomino 'I' configuration in the elementary cellular automaton rule 110, running in 5,000 steps. The periodic background is filtered in one colour for a better view of gliders and collisions

Research in progress reports a number of interesting reactions however, a glider gun is not discovered in this domain, moreover a glider gun in rule 22 with memory is reported in [18]. Thus, phenotypically we can see that rule 22 has a connection with the Game of Life.

In [9] Cook proposed that elementary cellular automaton rule 110 can be called as *LeftLife*. However, the relation between the Game of Life and rule 110 is only phenotypical because rule 110 is not tangential to the identity and it does not have unstable fixed points that equilibrate stable fixed points. Rule 110 increases close to double the probability to get states 1s in the next generation with respect to the Game of Life. The local rule φ is defined as follows:

$$\varphi_{R110} = \begin{cases} 1 \text{ if } 110, 101, 011, 010, 001 \\ 0 \text{ if } 111, 1000, 000 \end{cases} . \quad (2)$$

Also, rule 110 evolves with a periodic background and not a stable state. Which is interesting is that rule 110 has a glider gun and here it is possible to produce extensible glider guns between a diversity of rule 110 objects [19, 20]. From random initial conditions typically the attractors are dominated by gliders moving to the left ($\overline{E}$ gliders[3]). On the other hand, if we start a small configuration hence the rule 110 evolves always to the left. In Fig. 6 a pentomino 'I' is codified in the initial condition and it evolves during 3,000 generations before reaching its periodic pattern moving to the left, the frequency of gliders moving to the left is most of 80% and a barrier to the right prevents any perturbation coming to the left.

3 Final Notes

The Game of Life without question is the most studied cellular automaton, explored by a wider range and rich spectrum of researchers. The experts in Life constructions design very large and complex patterns to reach some limits of the rule. The number of complex patterns increases and a very specialized simulator is created to explore these huge spaces, such as *Golly* (http://golly.sourceforge.net/) where a number of sophisticated constructions have been designed. Golly is the most complete and powerful simulator to run Life and other cellular automata. What is the limit? When one of authors asked this question to Harold McIntosh in Puebla years ago, Harold responded: *well, the limit is the Ackermann function*.

A small repository about the Game of Life is accessible from https://www.comunidad.escom.ipn.mx/genaro/Cellular_Automata_Repository/Life.html.

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Genaro J. Martínez is a professor at the School of Computer Sciences, National Polytechnic Institute, Mexico City, Mexico and visiting fellow at the Unconventional Computing Centre, University of the West of England, Bristol, UK. He does research in cellular automata, unconventional computing, artificial life, complex systems, and swarm robotics.

Andrew Adamatzky is a professor at the Department of Computer Science and Director of the Unconventional Computing Centre, University of the West of England, Bristol, UK. He does research in reaction-diffusion computing, cellular automata, massive parallel computation, collective intelligence, bionics, complexity, non-linear science, novel hardware.

Juan C. Seck-Tuoh-Mora received the M.S. and Ph.D. degrees in computer science from the Center for Research and Advanced Studies, National Polytechnic Institute, Mexico, in 1999 and 2002, respectively. He is currently a Professor-Researcher of the academic area of engineering with the Autonomous University of the State of Hidalgo. His current research interests include cellular automata, metaheuristics, evolutionary algorithms, and neural networks to model, design, optimize, and control engineering systems.