

On Circular Growth of Life-Like Automata



Genaro J. Martínez, Andrew Adamatzky, Hector Zenil,
and Juan C. Seck-Tuoh-Mora

Abstract Growing circular patterns in nature is usually attributed to diffusion processes (e.g. chemical systems), multiplication of organisms (e.g. bacterial colonies) and diffusion-limited aggregation (e.g. crystal formation). The growth phenomenon in cellular automata was originally conceptualized by John von Neumann, highlighting the connection between excitable and resting states. We present a Life-like evolution rule capable of exhibiting growth similar to nearly perfect circular patterns, and performing such approximations better than previous rules by same automaton means. Results may be relevant to space-filling, numerical methods and packing applications.

Keywords Cellular automata · Chemical reaction · Growth · Circle · Collisions

G. J. Martínez (✉) · A. Adamatzky · H. Zenil · J. C. Seck-Tuoh-Mora
Computer Science Laboratory, Escuela Superior de Cómputo, Instituto Politécnico Nacional, Mexico City, Mexico
e-mail: gjuarezm@ipn.mx

A. Adamatzky
e-mail: andrew.adamatzky@uwe.ac.uk

H. Zenil
e-mail: hector.zenil@cs.ox.ac.uk

J. C. Seck-Tuoh-Mora
e-mail: jseck@uaeh.edu.mx

G. J. Martínez · A. Adamatzky
Unconventional Computing Laboratory, University of the West of England, Bristol, UK

H. Zenil
School of Biomedical Engineering and Imaging Sciences, King's College London, London, UK

J. C. Seck-Tuoh-Mora
Área Académica de Ingeniería y Arquitectura, Universidad Autónoma del Estado de Hidalgo, Pachuca, Mexico

1 Introduction

The concept of growth in cellular automata was considered by von Neumann from its origins in [1]. One of the goals was to develop efficient ways of propagating signals from excitable cells to resting cells in the universal constructor. A complementary objective was to understand how to control the growth preventing the growing pattern from transiting to chaos. In the present paper, we analyse a controllable growth in two-dimensional binary-state isotropic-neighbourhood semi-totalistic cellular automaton. Pioneer results simulating growth and nucleation with cellular automata were presented by Margulos, Toffoli and Vichniac in [2]. Gravner and Griffeath developed an extensive formalism and analytical approach to nucleation and growth in [3–5]. They also provided classifications of cellular automata based on type of their growth. Complimentary results on circular pattern growth could also be found in [6].

The macroscopic behavior of a system can be understood by observing the collective dynamics of thousands or even millions of elements, each with its own density, instead of focusing solely on the microscopic properties. This approach allows us to observe and analyze a wide range of phenomena, such as growth, nucleation, diffusion, and pattern formation, which arise in various scientific disciplines including physics, chemistry, biology, and cosmology in general [9]. Examples of such phenomena include the expansion of a chemical wave, crystal growth, pigmentation, virus evolution, population dynamics, reactions involving a large number of particles while preserving energy locally and increasing volume, propagation of waves in confined channels, and the expansion of a matter, see e.g. Fig. 1.

The paper is structured into three main sections. The first section provides an overview of the nomenclature and fundamental concepts related to cellular automata. The second and third sections delve into our experimental investigations, focusing on different cellular automata rules. We present examples illustrating phenomena such as growth, the emergence of simple waves, nucleation, and the collisions between these entities. Furthermore, we analyze the general properties of our experiments by conducting a comparative probabilistic analysis, incorporating the respective mean field theory polynomials [10, 11].

2 Previous Work, Nomenclature and Basic Notation

In their work [12], the authors proposed a comprehensive and phenomenological classification for two-dimensional cellular automata across a broad spectrum of semi-totalistic cellular automata functions, utilizing Bays' notation [13]. They discussed a taxonomy and examined rules that exhibit complex behavior, including those that generate intriguing patterns or demonstrate growth. Additionally, the paper explores the utilization of precipitating chemical systems to approximate Voronoi [14], the transmission of signals through channel-like structures to convey information or

enable particle-based self-localization [15], as well as the construction of patterns through specific initial conditions or particle propagation [12].

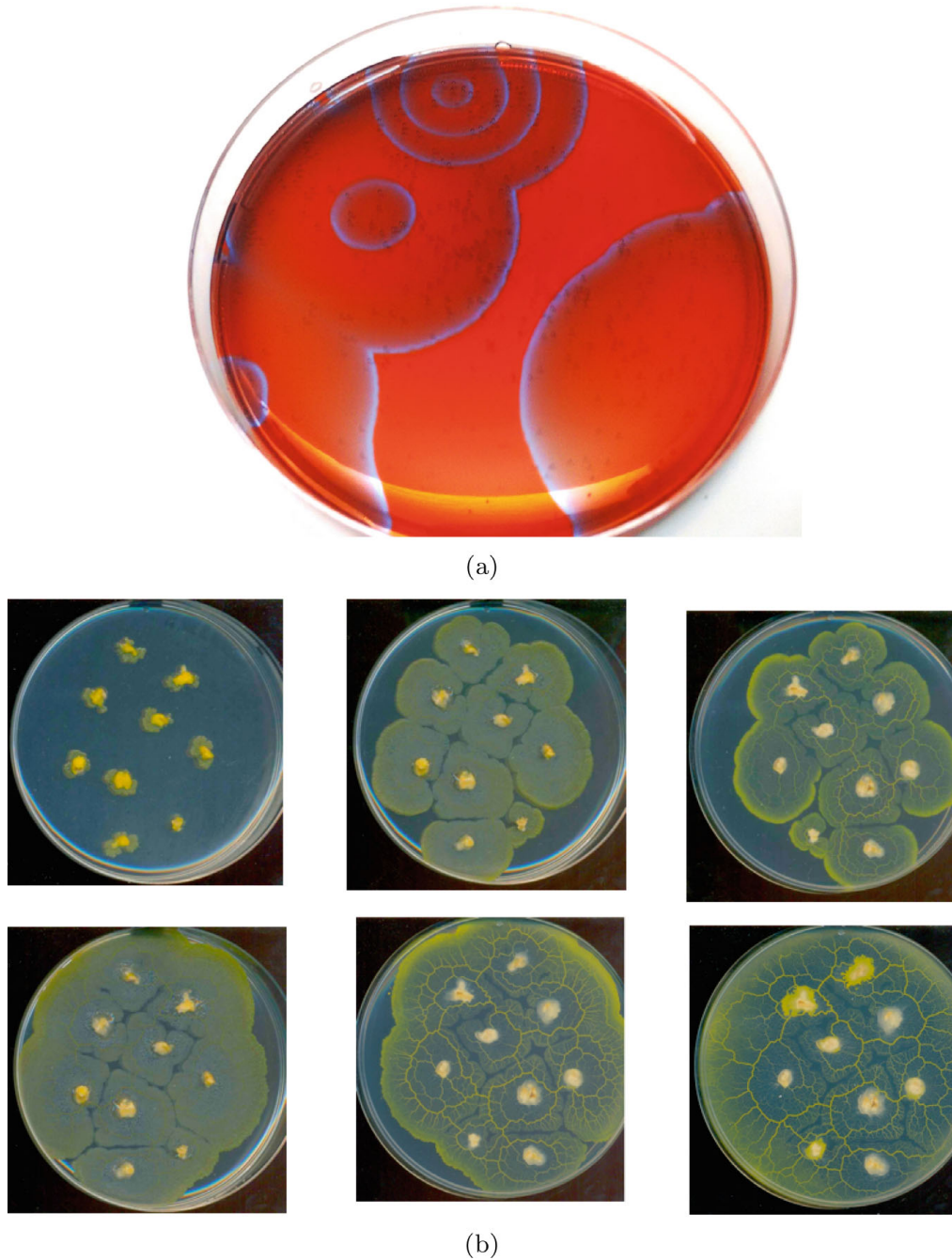


Fig. 1 **a** A snapshot of Belousov-Zhabotinsky medium, where oxidation wave-fronts are seen in blue. See more details in [7]. **b** Snapshots of the growth of slime mould *Physarum polycephalum* from several sites of inoculation. Snapshots are taken every 6 h from top left to bottom right. See more details in [8]

Numerous authors have reported various patterns and interesting rules in different orders of cellular automata [16–26] contributing to the body of research in this area. In [27, 28], pattern formation such as discrete Turing patterns by means of Turing machines were also explored.

Life-like rules are two-dimensional binary-state semi-totalistic cellular automata defined by a quadruple as $\langle \mathbb{Z}^2, \Sigma, u, f \rangle$. Where $\mathbb{Z}$ is the set of integers and every cell $x_{i,j} \in \mathbb{Z}^2$ has eight neighbours, orthogonal and diagonal (isotropic or Moore's neighbourhood) and $u(x) = \{y \in \mathbb{Z} : x \neq y \text{ and } |x - y| \leq 1\}$, $\Sigma = \{0, 1\}$ the alphabet, and f is a local transition function defined as follows:

$$x^{t+1} = f(u(x^t)) = \begin{cases} 1, & \text{if } (x^t = 0 \text{ and } \sigma_x^t \in [\theta_1, \theta_2]) \text{ or } (x^t = 1 \text{ and } \sigma_x^t \in [\delta_1, \delta_2]) \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

where $\sigma_x^t = |\{y \in u(x) : y^t = 1\}|$, and $\theta_1, \theta_2, \delta_1, \delta_2$ are some fixed parameters such that $0 \leq \theta_1 \leq \theta_2 \leq 8$ and $0 \leq \delta_1 \leq \delta_2 \leq 8$. Every cell in this 2D regular lattice is updated simultaneously each time.

The conventional notation can be extended to represent rules as two conditions $B\theta_1 \dots \theta_8 / S\delta_1 \dots \delta_8$. By using this notation, a subset of these rules can be understood as a discrete model of a quasi-chemical system with a substrate represented as 0' and a reagent as 1'. In this interpretation, the intervals $[\theta_1, \theta_2]$ denote the range of reactions or associations between the substrate and the reagent, while $[\delta_1, \delta_2]$ represents the range of dissociation. Notably, this family of rules includes Conway's Game of Life, where the intervals $[\delta_1, \delta_2]$ and $[\theta_1, \theta_2]$ correspond to the intervals of survival and birth, respectively. It is worth mentioning that this notation aligns with Bay's notation for certain Life-like rules, expressed as $R(B_{\min}, B_{\max}, S_{\min}, S_{\max})$ [13].

3 Growth-Like Waves in Semi-Totalistic Cellular Automata

In 1985, Packard and Wolfram conducted a study on various totalistic binary two-dimensional cellular automata, and Wolfram specifically identified an evolution rule capable of constructing a circle pattern (see Fig. 2). The details of this rule can be found in [6], where the emergence of the circle pattern is illustrated during different steps of evolution.

Figure 2 depicts the evolution of the Life-like rule $B3/S01234$, which demonstrates the ability to generate a circle pattern. The evolution commences with an initial configuration consisting of a horizontal line comprising 7 cells in state 1. Over the course of 73,206 generations, the pattern evolves and grows, resulting in a population of $1.0928157e+9$ cells in state 1. The pattern was simulated using the *Golly* software.¹ Figure 2a shows a symmetrical pattern at the center of the evolution,

¹ <https://golly.sourceforge.net>.

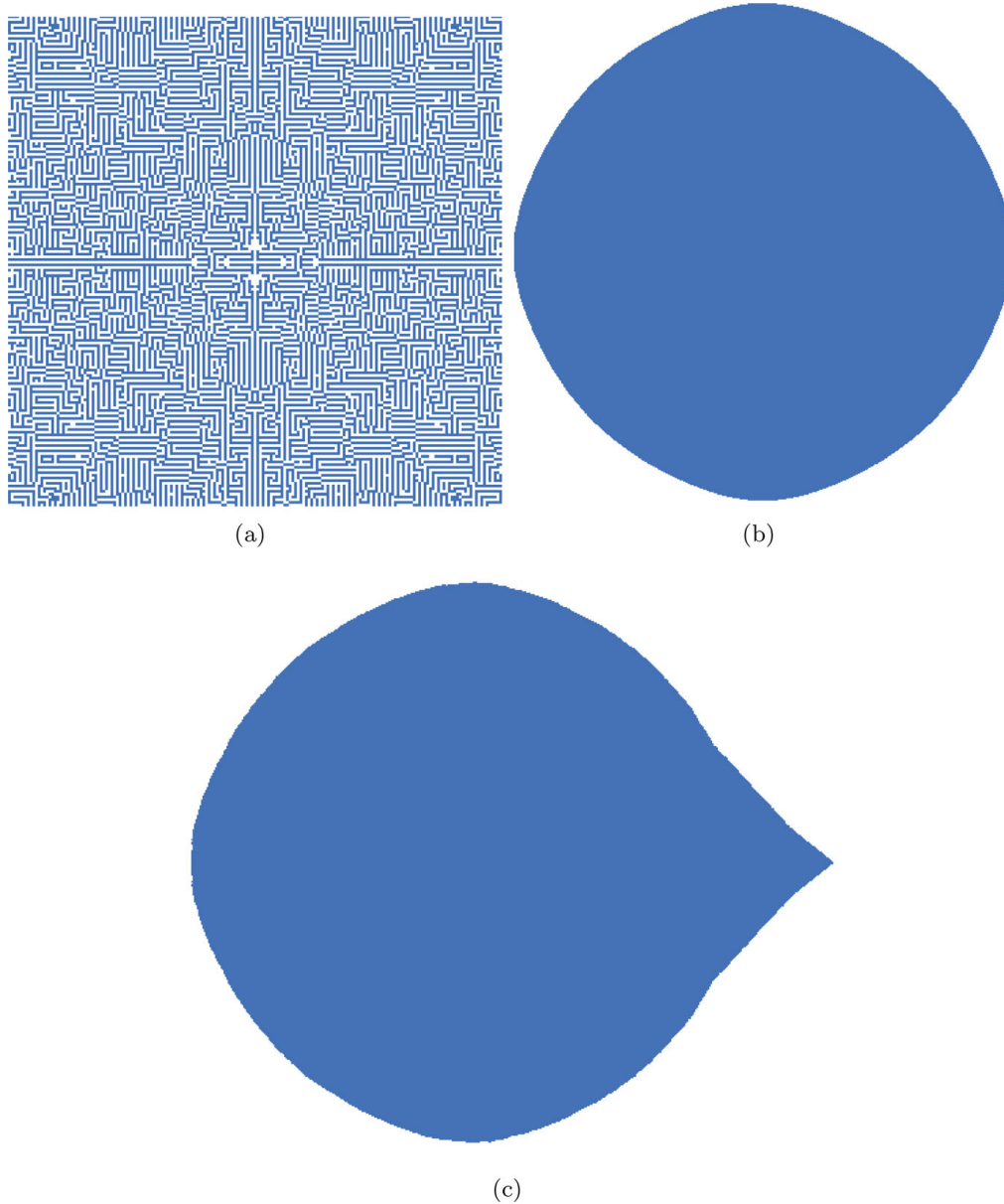


Fig. 2 Circular pattern constructed with the Life-like evolution rule $B3/S01234$. The initial configuration starts with a horizontal line of 7 cells in state 1. **a** Shows the central domain of evolution. **b** Shows the evolution after 73,206 generations with a population of $1.0928157\text{e}+9$ cells. **c** Displays the evolution of the initial string 10111 where the circular pattern cannot be produced, this evolution is calculated for 49,995 generations with a population of 2.8×10^8 (patterns calculated with Golly)

giving rise to labyrinthine forms. Figure 2b highlights the emergence of the circular pattern within the evolution space.

We introduce another evolution rule that exhibits similar capabilities to the previously discussed rule. The new rule is denoted as $B3/S234$ and demonstrates a high

degree of robustness against perturbations. It has been observed that Wolfram's rule, as discussed by Gray several years ago, is sensitive to minor changes from a specific initial condition, resulting in the destruction of the circle pattern and the emergence of a peak at an extreme point (Fig. 2c, see [29]).

However, we have discovered a method to rectify this issue and restore the circular pattern. By colliding two configurations with peaks positioned 180° apart and allowing them to evolve for 10^5 generations, the circle pattern is successfully recovered. Figure 3 illustrates different stages of the collision process, showcasing the fusion of the two peaks and the subsequent restoration of their circular form. Additionally, a close-up snapshot of the center of the collision is provided.

Regardless of the initial condition perturbations (mutations) applied, the Life-like rule $B3/S234$ consistently maintains its circular pattern with a high frequency. Table 1 displays the patterns associated with 1-bit and 2-bit mutations, where each string corresponds to a specific pattern and its probability to emerge.

Figure 4 depicts the growth of a circular pattern originating from a line of 11 cells in state 1, following the Life-like rule $B3/S234$. This evolution occurs over a span of 153,075 generations, resulting in a population of 4.17114×10^9 cells. In Fig. 4a, a static and symmetrical pattern is observed at the center of the evolution, reminiscent of the pattern produced by the rule $B3/S01234$, which gives rise to labyrinthine forms. Finally, Fig. 4b illustrates the circle pattern that emerges after 153,075 generations, although the circular form is attained within the initial 200 steps.

The evolution rules $B3/S01234$ and $B3/S234$ exhibit chaotic behavior when initialized with random conditions, where the evolution displays a competition between static patterns like labyrinths and undefined regions that gradually become stable over subsequent iterations. By visualizing the evolution in a three-dimensional projection achieved through the concatenation of two-dimensional planes, the regions of chaos become more apparent through their evolutionary history. Figure 5 illustrates this type of projection, where the initial condition starts with a line of seven cells in state 1 and evolves for 360 generations. Figure 5a shows the projection along the z -axis, while Fig. 5b presents the same evolution observed from the y -axis. These evolutions were calculated using the software tool *CAViewer*.²

Elements of chaos can be discerned through mean-field theory. Gutowitz and McIntosh employed this approximation to identify global behavior by considering the number of unfixed and fixed points over an extended period. Mean-field theory enables the description of the statistical properties of cellular automata without analyzing individual rule evolution spaces. This approach treats the elements of a set of symbols as independent and uncorrelated within the rule's evolution space. By studying the probabilities of states in the neighborhood in terms of the probability of a single state (i.e., the state to which the neighborhood evolves), one can analyze the probability of the neighborhood-state as a product of the probabilities of each cell-state in the neighborhood. This analysis yields a polynomial of probabilities, and the curve of this polynomial can be utilized to classify the rules, as proposed by McIntosh in [11] (Fig. 8).

² https://www.comunidad.escom.ipn.mx/genaro/Cellular_Automata_Repository/Software.html.

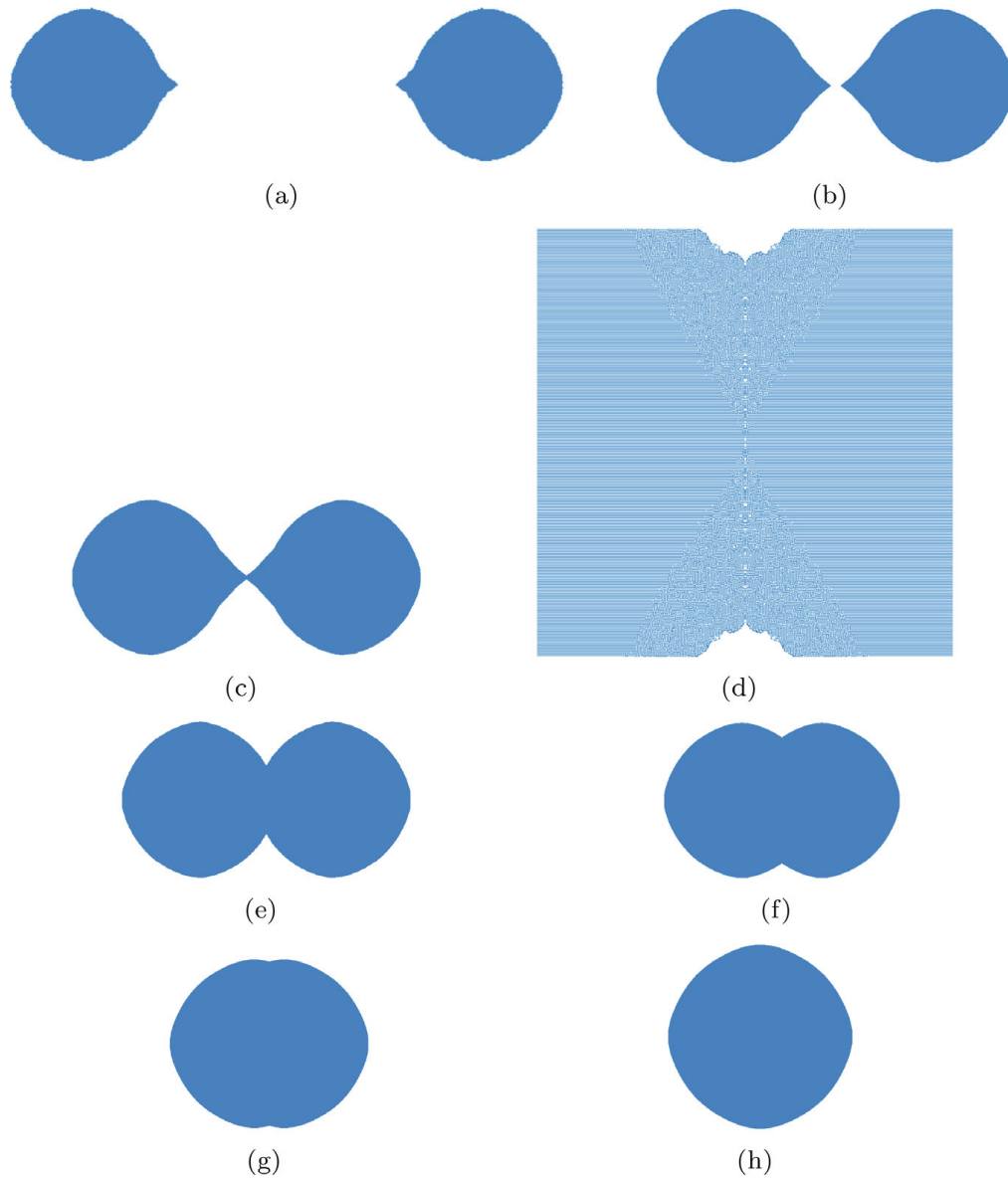


Fig. 3 A collision between two obtrusion patterns at 180° . The patterns are driven by the rule *B3/S01234*. The patterns recover their circular shape after 10^5 generations. The initial configurations for left and right circles are 11101 and 10111 respectively, then obtrusion patterns start to grow and eventually they collide at the center (shown in a close up). The colliding patterns merge to recover the circular pattern which eventually will grow without any other perturbation

Table 1 The evolution of various initial conditions in Life-like rules $B3/S01234$ and $B3/S234$ results in the formation of circular patterns. In the case of rule $B3/S01234$, we analyze lengths 5, 6, and 7 while for rule $B3/S234$, we analyze lengths 7 and 11. These initial conditions undergo mutation in one or two cells, ultimately leading to the emergence of circular patterns in the respective rules. Most high rates where a circle can emerge in this domain is 43% for a length 6 and 45% for a length 11 respectively. However for length 7 the probability is 64% considering circles and semi-circles, and 54% for the second rule

$B3/S01234$		$B3/S234$	
string	evolution	string	evolution
11111	Circle	1111111	Circle
10111	Peak	1011111	Disappear
11011	Still life	1101111	Still life
10011	Still life	1110111	Two oscillators
10101	Still life	1001111	Still life
111111	Circle	1100111	Oscillator
101111	Circle	1101011	Disappear
110111	Circle	1101101	Disappear
100111	Semi-circle	1110101	Oscillator
110011	Still life	1011101	Oscillator
101011	Still life	1000111	Oscillator
101101	Still life	1001001	Disappear
1111111	Circle	1111111111	Circle
1011111	Semi-circle	1011111111	Circle
1101111	Semi-circle	1101111111	Circle
1110111	Semi-circle	1110111111	Circle
1001111	Semi-circle	1111011111	Circle
1010111	Semi-circle	1111101111	Circle
1011011	Still life	1001111111	Circle
1011101	Still life	1010111111	Circle
1100111	Semi-circle	1100111111	Circle
1101011	Still life	1101011111	Disappear
1101101	Still life	1110011111	Oscillator
		1110101111	Oscillator
		1111001111	Still life
		1111010111	Two still life
		1110110111	Oscillator + still life
		1101110111	Oscillator + still life
		1011110111	Two still life
		1011101111	Semi-circle
		1011011111	Disappear
		1011111011	Semi-circle
		10111111011	Disappear
		10111111101	Circle

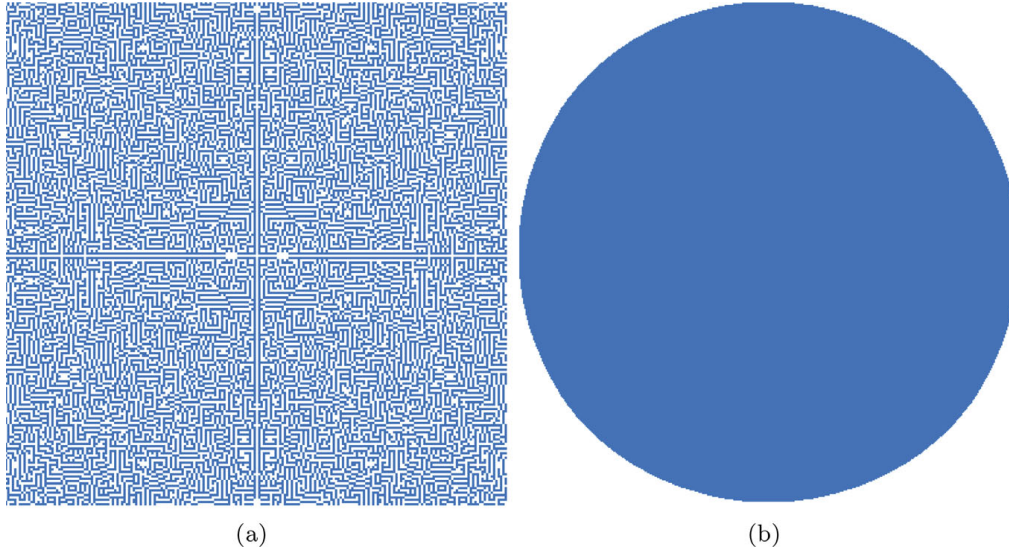


Fig. 4 Using the Life-like evolution rule $B3/S234$, a circular pattern is formed. The initial configuration consists of a line containing 11 cells in state 1. In **a**, the center of the evolution is displayed. Figure **b** presents the evolution after 153,075 generations, resulting in a population of $4.17114\text{e}+9$ cells

Mean field polynomial defined by the evolution rule $B3/S01234$ is:

$$p_{t+1} = p_t q_t^8 + 8p_t^2 q_t^7 + 84p_t^3 q_t^6 + 56p_t^4 q_t^5 + 70p_t^5 q_t^4 \quad (2)$$

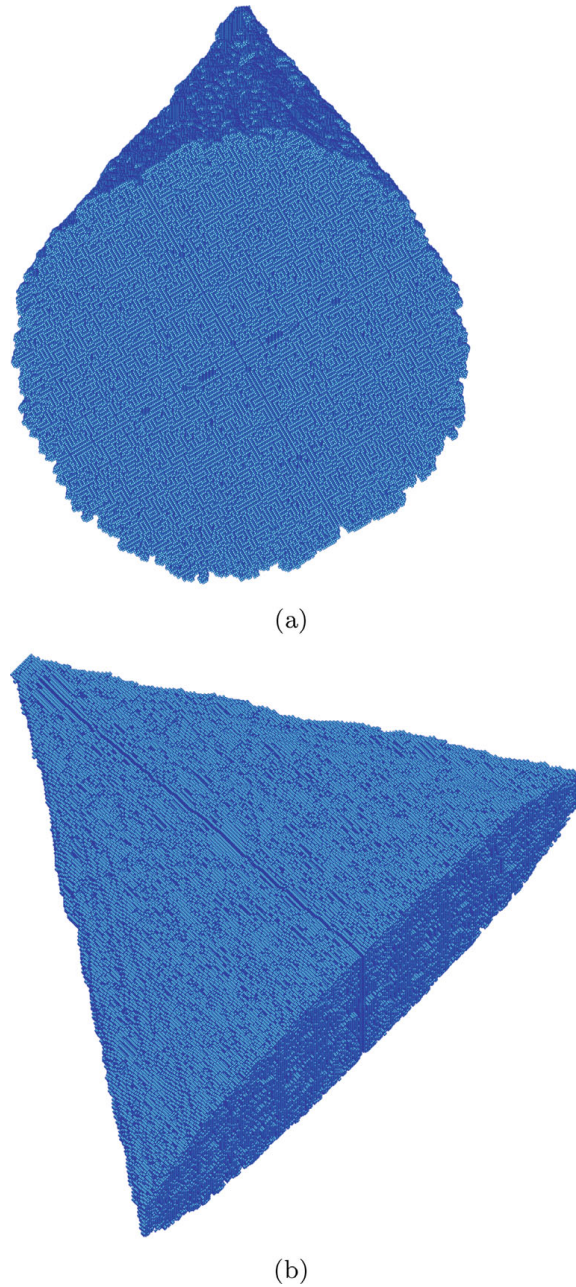
where some unstable fixed points overlap the identity function in the interval 0.015 to 0.028 while a stable point is located in 0.463. Unstable fixed points display the emergence of islands that grow quasi-circular but they are perturbed by an ample number of still-life configurations that are highly frequent starting from these densities (see Fig. 6a, b). While the stable fixed point determines the final average density for a long time, thus the final density is reached in very few dozens of steps (see Fig. 6c, d).

Mean-field polynomial defined by the evolution rule $B3/S234$ is:

$$p_{t+1} = 84p_t^3 q_t^6 + 56p_t^4 q_t^5 + 70p_t^5 q_t^4 \quad (3)$$

where an unstable fixed point is localized in 0.183. However, exploring initial conditions starting at this value does not show interesting behaviour for exploring some kind of nucleation; it will start with densities smaller than 0.03 which is close to the fixed point at the origin. On these values, the state 0 is dominant with few still life where some small perturbations begin an irreversible expansion (see Fig. 7a, b). While the stable fixed point is localized in 0.447, thus starting random initial conditions in this value reach its average population in a few steps (see Fig. 7c, d).

Fig. 5 Using the Life-like evolution rule $B3/S234$, a circular pattern is created by concatenating two-dimensional planes to obtain a three-dimensional representation. The initial configuration consists of a line of 7 cells in state 1, evolving over 360 generations. This projection provides a visualization of the evolutionary history characterized by chaotic behavior, with identifiable non-stable regions present



Both curves for $B3/S01234$ and $B3/S234$ across the diagonal and are not tangential with respect to the identity, this property implies that both rules relate chaotic global behaviour.

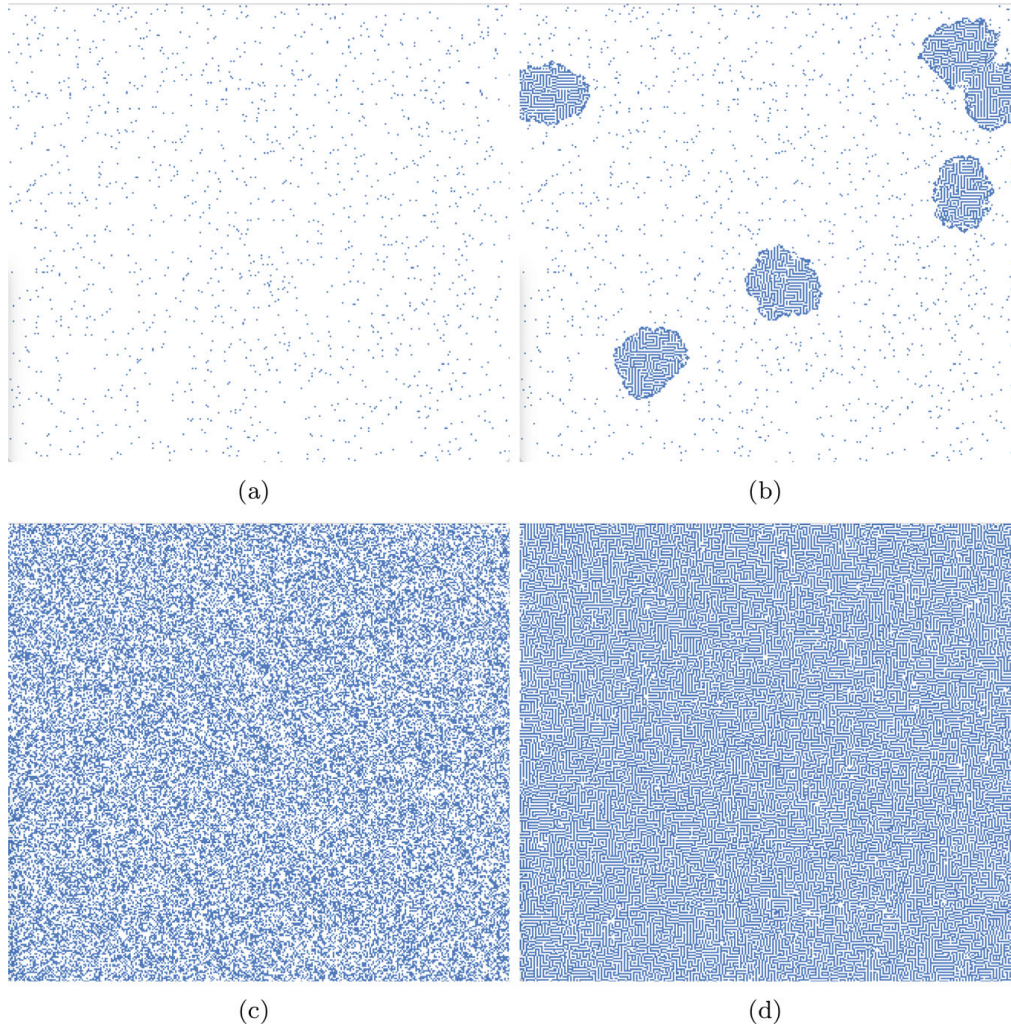


Fig. 6 Value fixed points as initial conditions in Life-like rule $B3/S01234$

4 Conclusions and Future Work

In this chapter, we present a distinct Life-like cellular automaton, namely rule $B3/S234$, which possesses the capability to generate a circular pattern. So this rule, complimentary to Wolfram's rule, exhibits a best preservation of the circular form and greater robustness against perturbations (mutations) from its initial condition, see Fig. 9. Both rules emerge with a similar construction resulting from chaotic global behavior. However, Wolfram's rule, when initiated with low densities, demonstrates a nucleation reaction wherein an explosion of chaos ensues. This particular growth pattern is swiftly influenced by a substantial number of still-life patterns that await activation (Fig. 6). Nevertheless, the rule $B3/S234$ evolves on very large stable regions dominated by the state 0 and the nucleation phenomenon start with few active cells (Fig. 7).

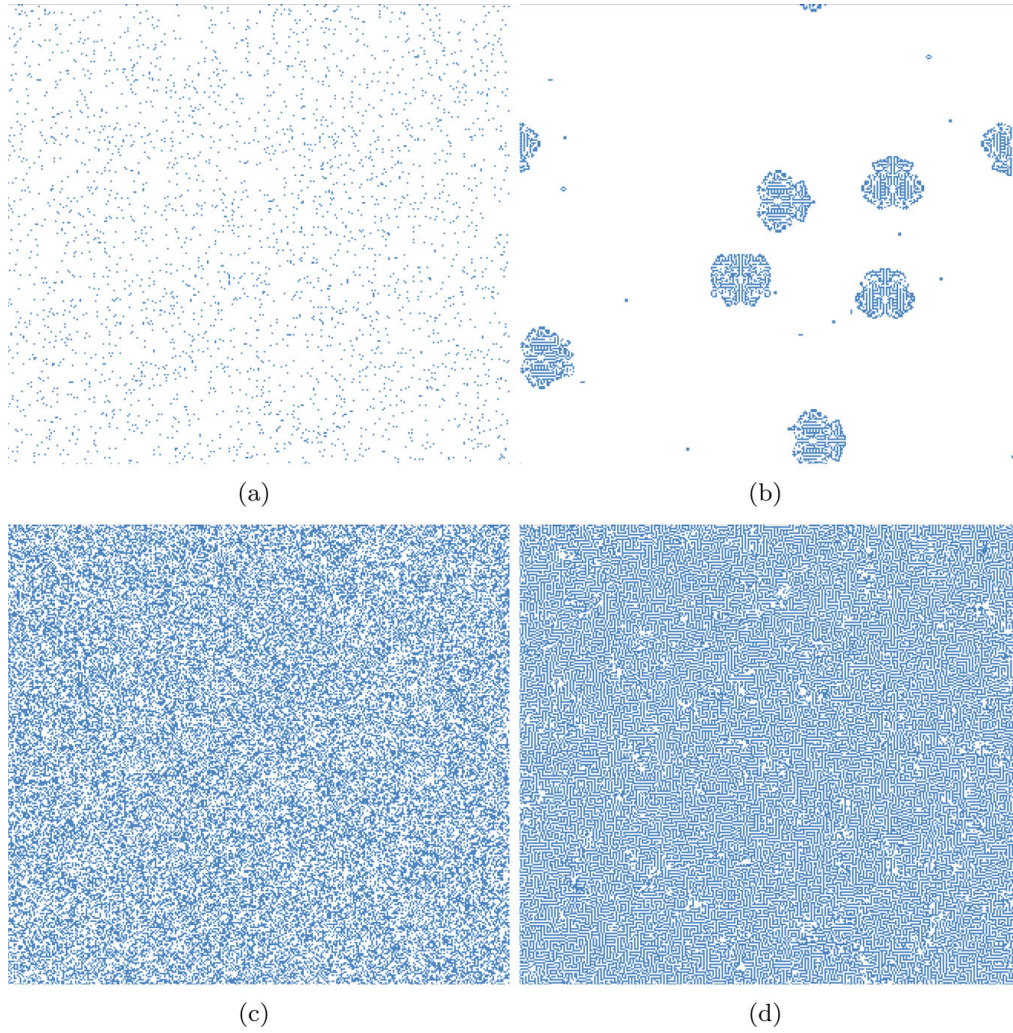


Fig. 7 Value fixed points as initial conditions in Life-like rule $B3/S234$

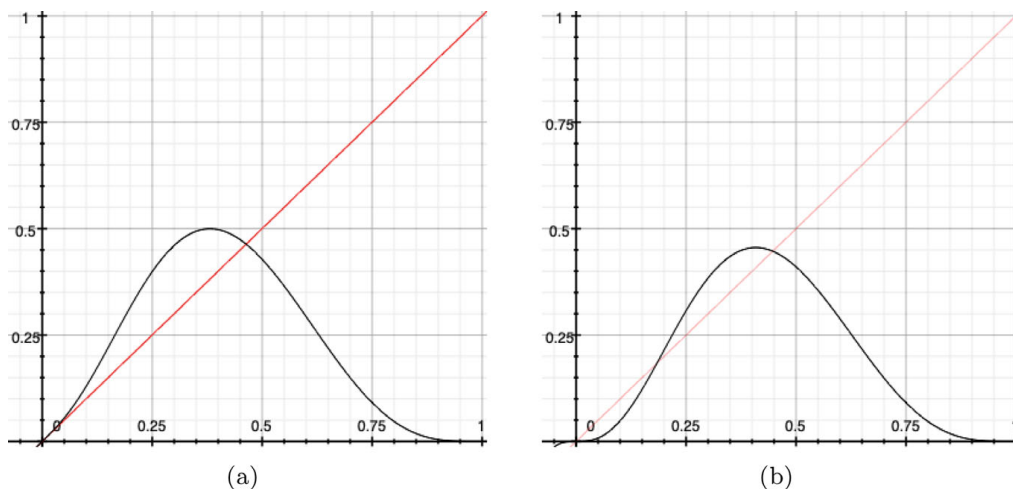


Fig. 8 Mean field curves for Life-like rules **a** $B3/S01234$ and **b** $B3/S234$

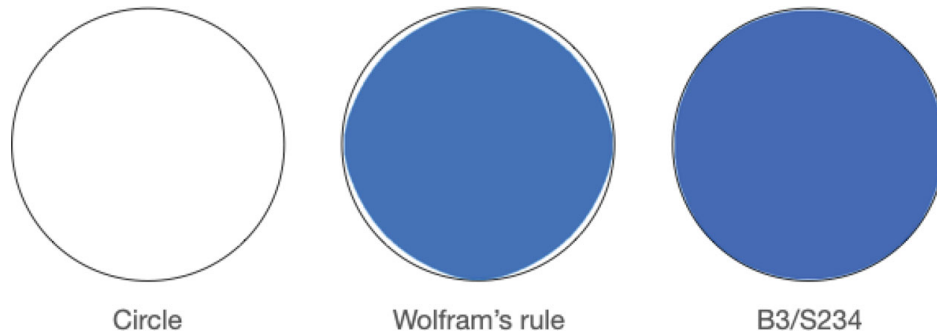


Fig. 9 This figure shows the cellular automata approximation to construct a circle. The left figure is a typical perfect circle drawn with computer software. The center figure is the circle filling approximation using Wolfram's rule and, the right figure shows the circle filling calculus with the function $B3/S234$ introduced in this paper

We believe that an application of this kind of work can be to space-filling and packing problems where smaller objects have to be optimally distributed in larger containers. Also relevant to problems of simulation and numerical methods for space filling algorithms such as computing rendering or applications to micro and fluid dynamics. Generalisations to three dimensions are possible by expanding the search space to a fourth dimensional construction rule space to fill three dimensional spaces.

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