CA Life rule
B2/S2345678

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Computation by competing patterns: Life rule $B2/S2345678$

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Bristol, England, UK
June 14, 2008

Abstract

Patterns, originated from different sources of perturbations, propagating in a precipitating chemical medium do usually compete for the space. These perturbations sub-divide the medium into unique regions given an initial configuration of disturbances. This sub-division can be expressed in terms of computation. We adopt an analogy between precipitating chemical medium and semi-totalistic binary two-dimensional cellular automata, with cell-state transition rule $B2/S2...8$. We demonstrate how to implement basic logic and arithmetical operations (computability) by patterns propagating in geometrically constrained Life rule $B2/S2...8$ medium.

The Game of Life CA

Conway's Game of Life [Gar70] (Life) is the best-known example of a universal collision-based CA. Its universality can be discussed by some ways, from its original demonstration developed by Conway *et al.* constructing a register machine [Con82], or from the construction of large and complicated Turing machine simulators constructed by Rendell [Ren02].

Martin Gardner

Mathematical Games — The fantastic combinations of John H. Conway's new solitaire game Life

Scientific American **223**, 120–123, 1970.

Elwyn R. Berlekamp, John H. Conway and Richard K. Guy

Winning Ways for your Mathematical Plays

Academic Press, 1982. (vol. 2, chapter 25)

Paul Rendell

Turing universality of the game of life

in *Collision-Based Computing*, Adamatzky (Ed.), Springer, 2002.

The Game of Life CA

The domain of Life is a regular two-dimensional lattice, a binary alphabet and a semi-totalistic local transition rule. The main characteristic of a semi-totalistic functions is to evaluate the central cell $x_{i,j}$ and depending of its value at time t and the sum of their eight neighbors, this value change or not at the next time $t + 1$. Thus a formal representation for this notation is:

$$\varphi(\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_\mathcal{V}) = \begin{cases} 1 & \text{if} \quad \begin{cases} \mathbf{x}_0 = 0 & \text{and} \quad B_{min} \leq \sum\limits_{i=1}^{\mathcal{V}} \mathbf{x}_i \leq B_{max} \\ \\ \mathbf{x}_0 = 1 & \text{and} \quad S_{min} \leq \sum\limits_{i=1}^{\mathcal{V}} \mathbf{x}_i \leq S_{max} \end{cases} \\ \\ 0 & \text{in other case} \end{cases}$$

$$(1)$$

Let $\Sigma = \{0, 1\}$ be the set of states, $\mathcal{V}$ the isotropic neighborhood, $\mathbf{x}_0 = x_{i,j}$ the central cell and, $\mathbf{x}_1, \ldots, \mathbf{x}_\mathcal{V} = x_{i-1,j-1}, \ldots, x_{i+1,j+1}$ their neighbors cells. Also we have two intervals with a minimum and maximum values: B_{min}, B_{max}, S_{min}, S_{max}; where B and S represent two actions: 'birth' and 'survival' respectively.

The Game of Life CA

Thus we have two representations to enumerate semi-totalistic rules.

1. Most traditional representation in Life community [Epp] is: $B0\ldots8/S0\ldots8$.

2. A notation proposed by Bays [Bay87] studying three-dimensional CA related to Life is: $R(S_{min}, S_{max}, B_{min}, B_{max})$.

This way, Life was represented simply as $B3/S23$ or $R(2333)$.

📄 David Eppstein
Gliders in Life-Like Cellular Automata (database)
http://fano.ics.uci.edu/ca/

📄 Carter Bays
Candidates for the Game of Life in Three Dimensions
Complex Systems **1**, 373–400, 1987.

Universal computing binary 2D CA

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minimal 2D CA	neighborhood	universality (or reference)
full	2×2	Powley [2008]
totalistic	von Neumann	unknown
semi-totalistic	von Neumann	unknown
full	von Neumann	Banks [1970]
totalistic	Moore	unknown
semi-totalistic	Moore	Life [1982]
		Life Turing [2000]
		Life Universal Computer [2002]
full	Moore	R rule [2006]

Table: Universal binary 2D CA (based in glider reaction).

We show a relation of universal 2D CA. Table shows kinds of local function (first column), kind of neighborhood (second column), and the specific CA (or reference) proved to be universal computing (third column). Also, almost all of them were constructed by glider-based collisions.

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State of results

Objectives

I Our goal is to show universal computing in other binary 2D CA Life rule.

II The construction will be developed by means of collisions from patterns propagation and not only based in gliders [Tof87].

III The implementation will be reached on a chaotic CA, representing class III in Wolfram's classification [Wol94].

📖 Tommaso Toffoli and Norman Margolus
Cellular Automata Machines
The MIT Press, Cambridge, Massachusetts, 1987.

📖 Stephen Wolfram
Cellular Automata and Complexity
Addison-Wesley Publishing Company, 1994.

Life rule $B2/S2\ldots8$

Life rule $B2/S2\ldots8$ is a binary two-dimensional CA.
The Life rule $B2/S2\ldots8$ is described as follows; each cell
takes two states '0' ('dead') and '1' ('alive'), and updates its
state depending on its eight closest neighbors as follows:

1. Birth: a central cell in state 0 at time step t takes state
   1 at time step $t+1$ if it has exactly two neighbors in
   state 1.

2. Survival: a central cell in state 1 at time t remains the
   same at time $t+1$ if it has more than one live neighbor.

3. Death: all other local situations.

Dynamics in Life rule $B2/S2\dots8$

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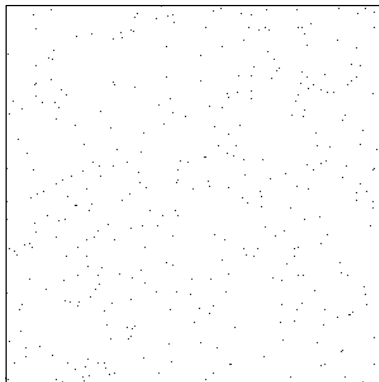Dynamics in Life rule
$B2/S2\dots8$
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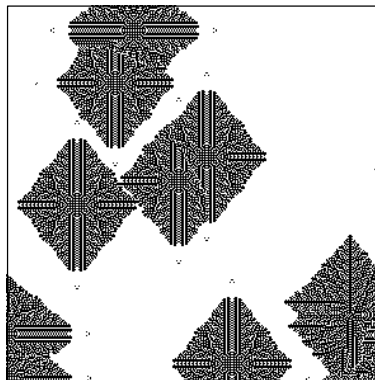Constructing a binary
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(a)

(b)

Figure: Snapshots showing activity of gliders and oscillators in $B2/S2\dots8$. (a) initial condition with a density of 0.004 into a lattice of 300×300. (b) evolution after 65 steps as nucleation phenomenon covering quickly the evolution space with a population of 23,802 live cells, and even some gliders and oscillators survival in this time.

Basic periodic structures in Life rule $B2/S2\ldots8$

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Looking for easy constructions, we can remember that a complex CA supporting gliders can be related as an interpreter of another CA [Boc91]. For instance, Rule 110 has a family of 12 gliders, Rule 54 has a family of four gliders, Life has a huge number of gliders, and so on.
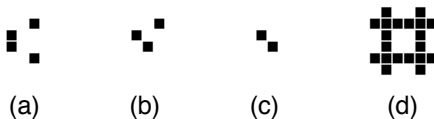

(a)   (b)   (c)   (d)

Figure: Basic periodic structures in $B2/S2\ldots8$: (a) glider period one, (b) oscillator period one, (c) flip-flop, and (d) still life configuration.

📄 N. Boccara, J. Nasser and M. Roger

Particle like structures and their interactions in spatio-temporal patterns generated by one-dimensional deterministic cellular automaton rules

Physical Review A **44 (2)**, 866–875, 1991.

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Important elements

A relevant characteristic is that rule *B*2/*S*2 . . . 8 supports *indestructible patterns*, which cannot be destroyed from any perturbation, they belong to the class of stationary localizations or still lives [Coo03,Mc88]. Its importance in Life community is for stopping explosions in Life rules and containing data as well. The minimal indestructible pattern is shown in the previous figure with its respective still live.

📄 Matthew Cook
Still Life Theory
in *New constructions in cellular automata*, Griffeath and Moore (Eds.), Oxford University Press, 2003.

📄 Harold V. McIntosh
Life's Still Lifes
http://delta.cs.cinvestav.mx/~mcintosh, 1988.

Indestructible pattern in $B2/S2\ldots8$

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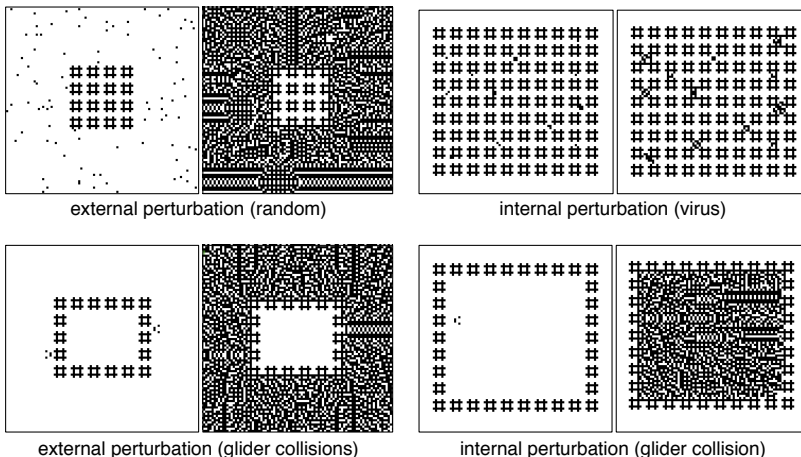Constructing a binary
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external perturbation (random)

internal perturbation (virus)

external perturbation (glider collisions)

internal perturbation (glider collision)

Figure: Indestructible Still Life colonies 'tested' by internal and external perturbations. Each pair of snapshots represents an initial condition (on the left) and a final, i.e. stationary, configuration (on the right).

Processing information

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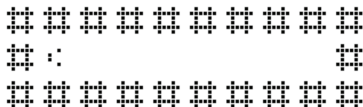$B2/S2 \ldots 8$
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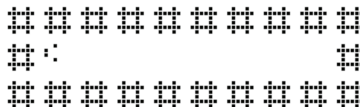Constructing a binary
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(a)   (b)

Figure: Feedback channels [Mor96] constructed with still life patterns from a glider stimulation (a) Initial state with the empty channel, a glider (top) and a final state representing value 0 (low), (b) Non-symmetric patterns representing value 1.

Kenichi Morita and Katsunobu Imai
Self-reproduction in a reversible cellular space
Theoretical Computer Science **168 (2)**, 337–366, 1996.

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Logic gates with a T-junction system

We implement computation with pattern propagation in the Life rule $B2/S2\ldots8$ as follows. A computing scheme is build as channels, geometrically constrained by Still Life indestructible blocks, and T-junctions (T-junction based control signals were suggested also in von Neumann [von66] work), between the channels. Each T-junction consists of two horizontal channels A and B (shoulders), acting as inputs, and a vertical channel, C, assigned as an output.
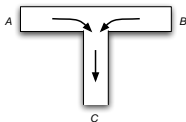

Figure: T-junction system.

📚 John von Neumann

Theory of Self-reproducing Automata (edited and completed by A. W. Burks)

University of Illinois Press, Urbana and London, 1966.

Logic gates with a *T*-junction system

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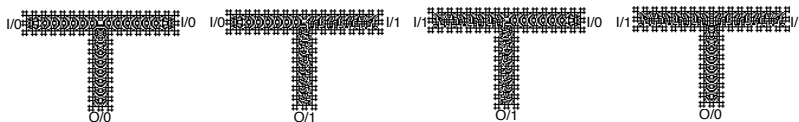Constructing a binary
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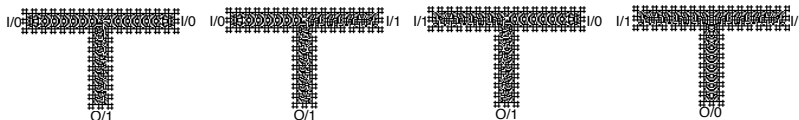
Figure: Implementing logic gates, two examples: NAND and XOR gates [Ada05]. Input binary values A and B are represented for In/0 or In/1, output result C is represented by Out/0 or Out/1.

📕 Andrew Adamatzky, Ben De Lacy Costello and Tetsuya Asai
Reaction-Diffusion Computers
Elsevier, 2005.

Half adder implementation

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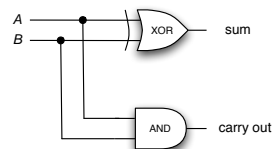CA

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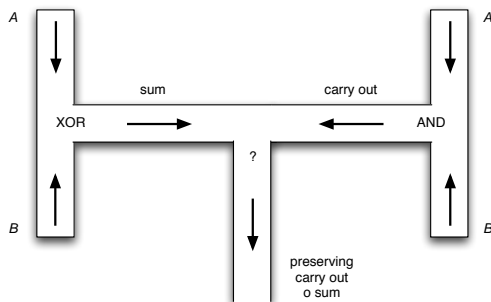adder

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Figure: Half adder circuit (left) and scheme of its implementation by pattern propagation in a geometrically constrained medium (right).

Half adder implementation

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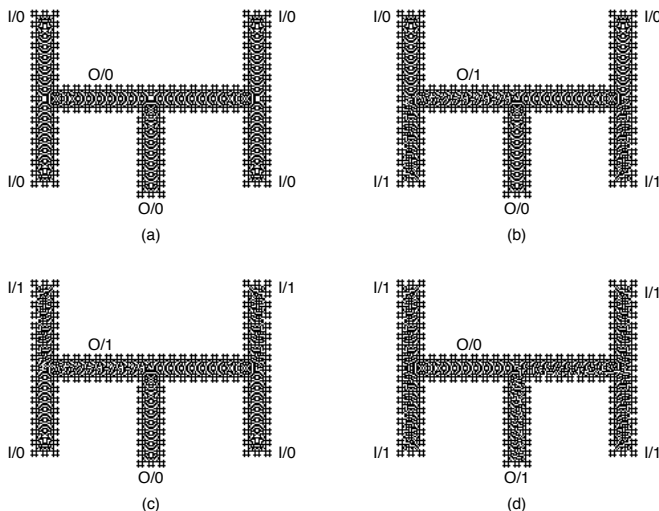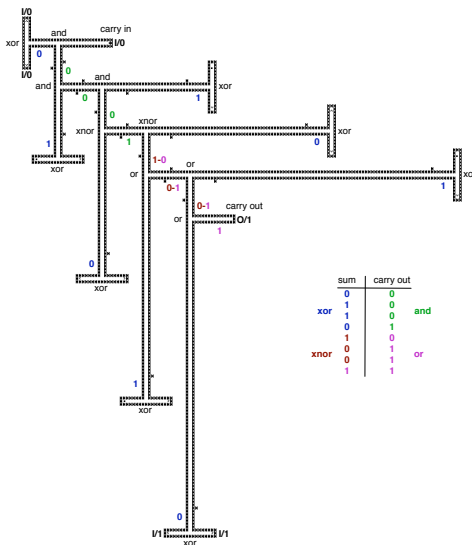adder

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Figure: Half adder implemented in Life rule $B2/S2...8$.
Operations represent sums (a) $0 + 0$, (b) $0 + 1$, (c) $1 + 0$, and (d)
$1 + 1$ where besides its carry out is preserved in this case.

Full adder implementation

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Figure: Full adder description of stages in Life rule $B2/S2\ldots8$ (show simulation).

Full adder implementation

The full binary adder consists of 18 gliders, 16 T-junctions, linked together by channels, and synchronization of signals as well. Finally the full adder is constructed on $1,118 \times 1,326$ cell lattice (1,482,468 cells), and there are 66,630 live cells evolving in 952 generations.

Conclusions

► The result presented in the paper demonstrates how to implement computation at the systems with pattern propagation based in T-junction systems. Also, we have shown universal computing in the Life rules domain with the Life rule $B2/S2...8$ by implementing a full binary adder. The relevance of this result in CA literature, specially into Life rules, is to prove universality in another evolution rule different from Life.

► Future work will concern with explicit construction of a Turing machine, solitons, conservative logic, and detailed study and classification of indestructible still life patterns in Life $dc22$ with two-dimensional de Bruijn diagrams.

► Sources, stuff and specific initial condition (.rle files) to reproduce these results are also available from: http://uncomp.uwe.ac.uk/genaro/diffusionLife/ B2-S2345678.html

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