

Collision-based Computing with Cellular Automata

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It is a project worked and shared by several years

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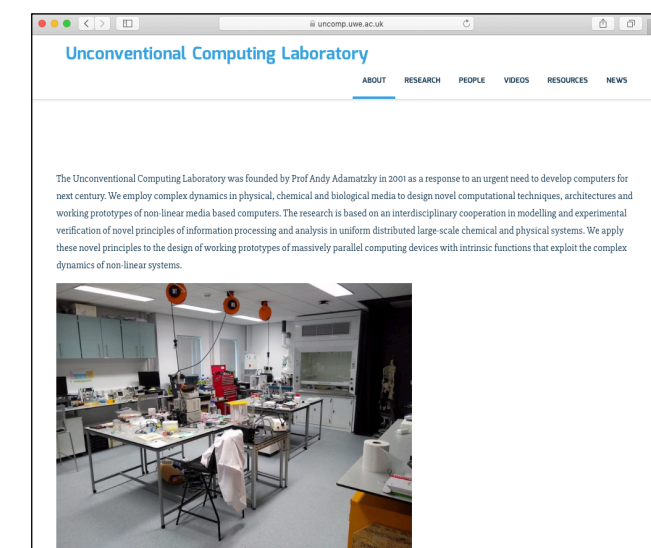
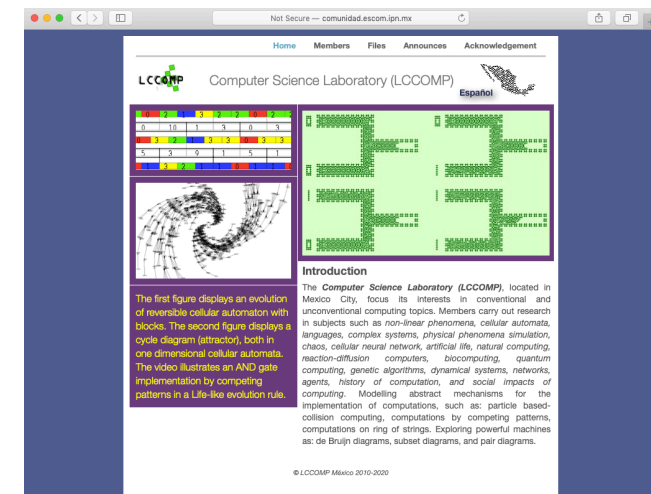
<https://zelinkaivan65.wixsite.com/navy/>

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Collaborations (researchers and students) and sponsors



Harold McIntosh



Andrew Adamatzky



Juan Seck



Shigeru Ninagawa



Matthew Cook



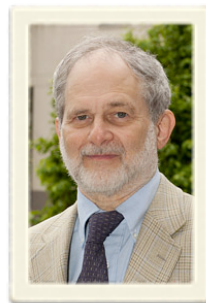
Andrew Wuensche



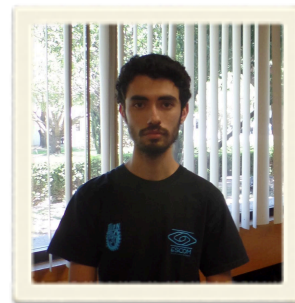
Stephen Wolfram



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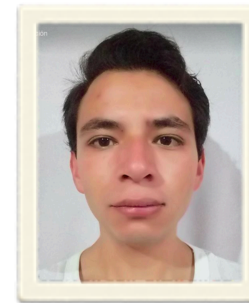
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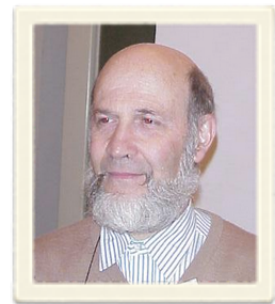
José Amador



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Guanrong Chen



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Christopher Stephens

Complex cellular automata a way to unconventional computing

Today, a “computer”, without further qualifications, denotes a rather well-specified kind of object; we’ll consider a computer “non-conventional” if its physical substrate or its organization significantly depart from this de facto norm. [Toffoli 1998]

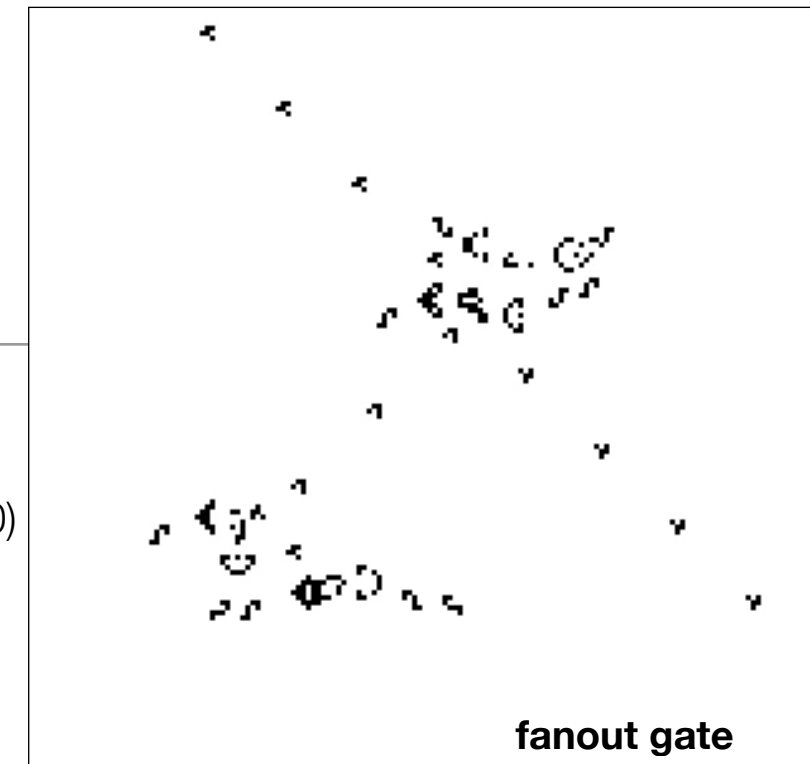
Unconventional computers shall exploit molecular computing level, to increase the power of computation, velocity, and storage. Actually, the term about of *unconventional computing* or *natural computing* have a number of directions:

- quantum computing
- DNA computing
- reaction-diffusion computing
- reversible computing
- tiling (pattern) computing
- origami computing
- Pysarum computing
- swarm computing

Antecedents and objectives

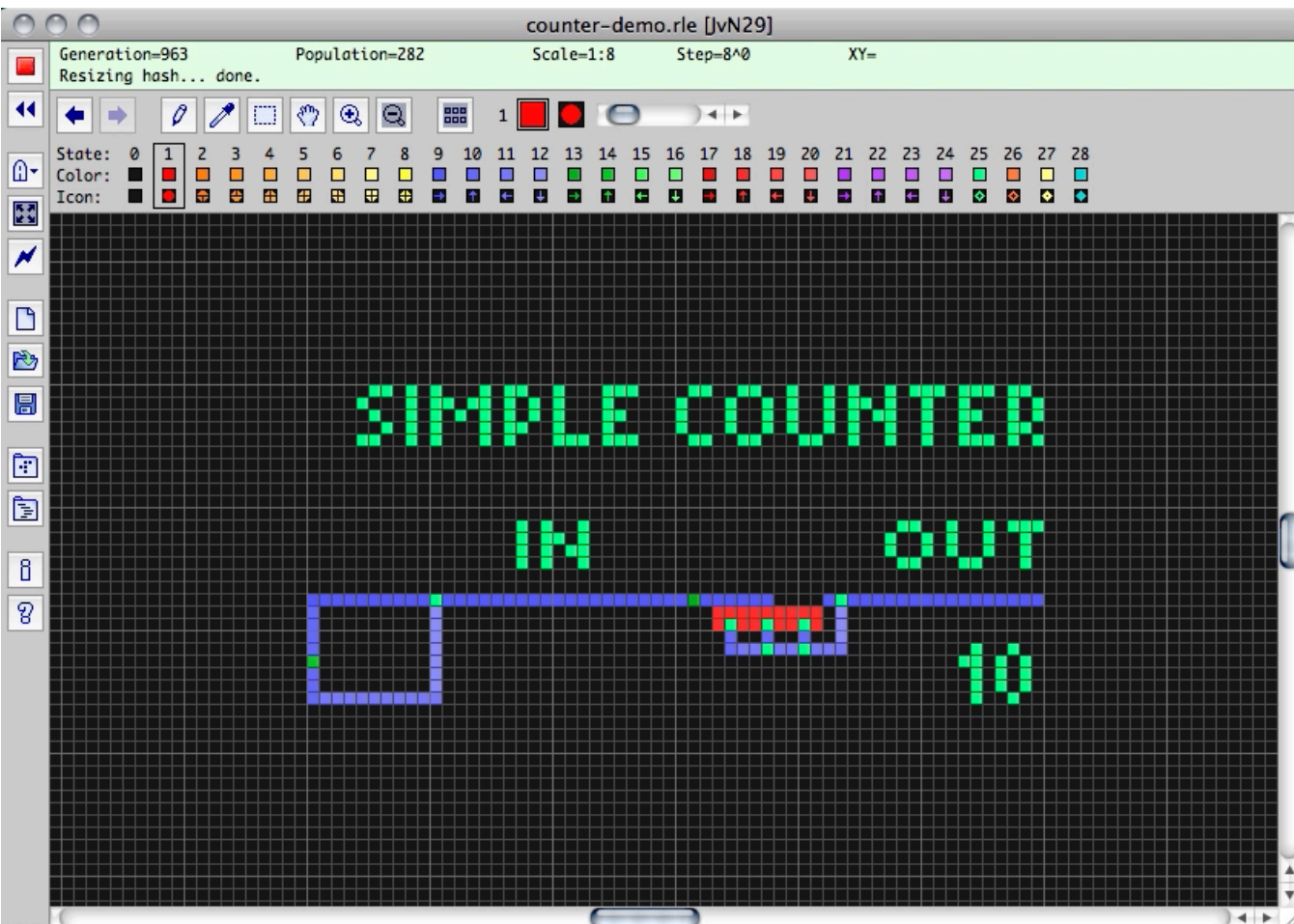
- Historically complex systems are related to computations.
- We explore a framework to design cellular automata computers on *colliders* and *cyclotrons*, a *circular way*.
- Basically, we work with complex elementary cellular automata.

Some samples of computations in cellular automata

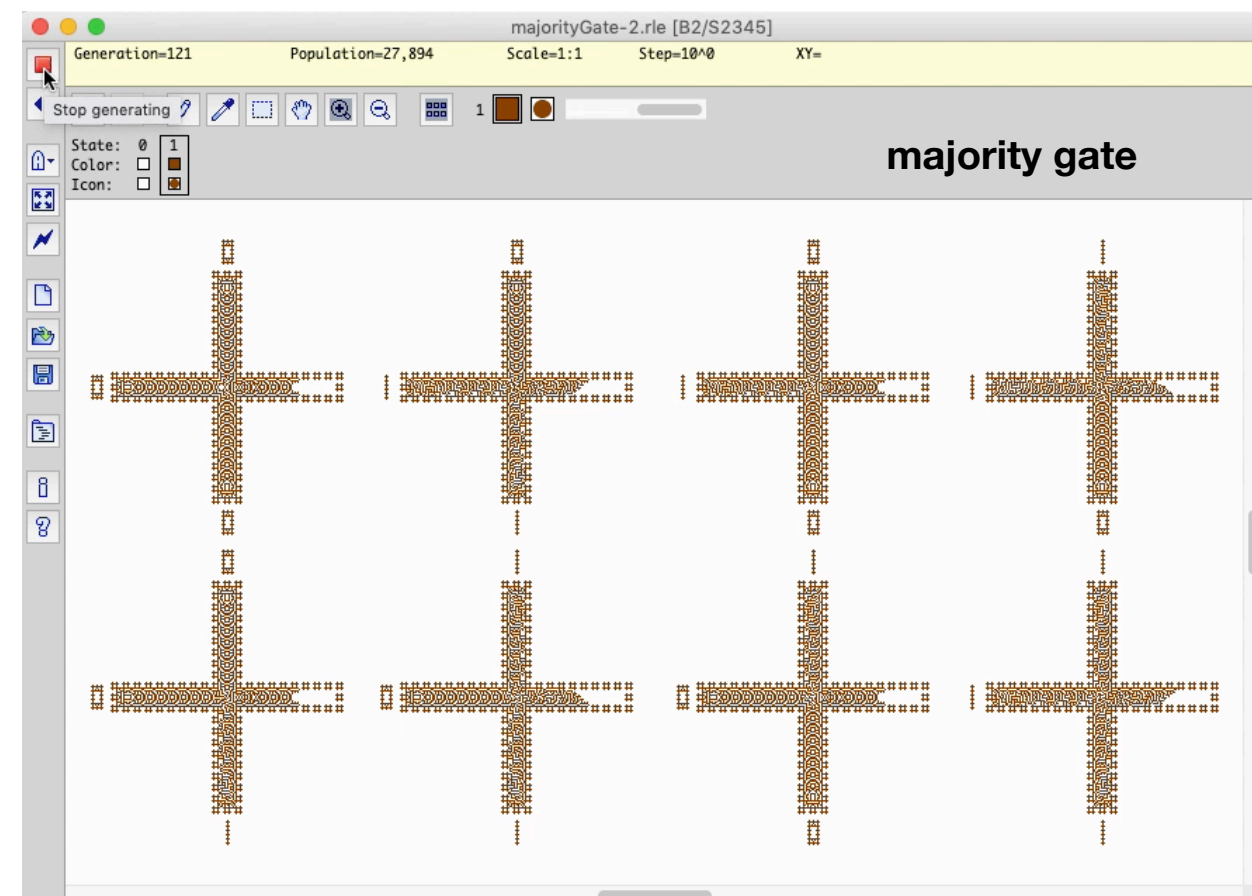


Game of Life
2-states CA (1970)

fanout gate



von Neumann 29-sates CA (1966)

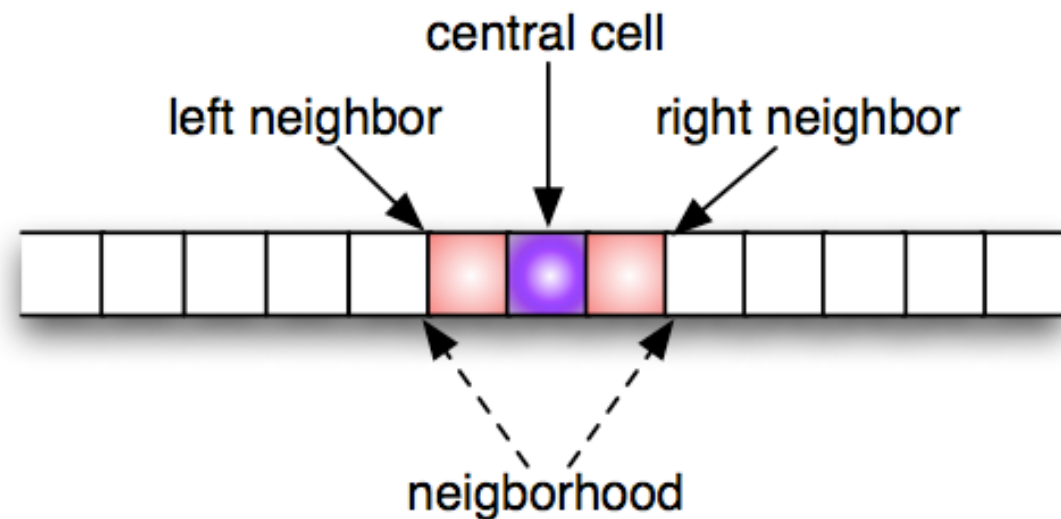


B2/S2345 29-sates CA (2008)

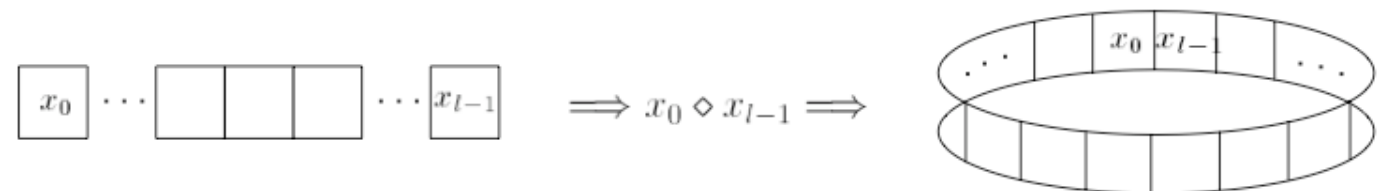
- By signals
- By gliders collision
- Propagation patterns

- Neumann, J. (1966) *Theory of Self-reproducing Automata* (edited and completed by A. W. Burks), University of Illinois Press, Urbana and London.
- Rendell, P. (2016) *Turing machine universality of the game of life*. Springer International Publishing.
- Martínez, G.J., Adamatzky, A., Morita, K. & Margenstern, M (2010) "Computation with competing patterns in Life-like automaton." In: Game of Life Cellular Automata, pp. 547-572. Springer, London.
- Golly <http://golly.sourceforge.net/>

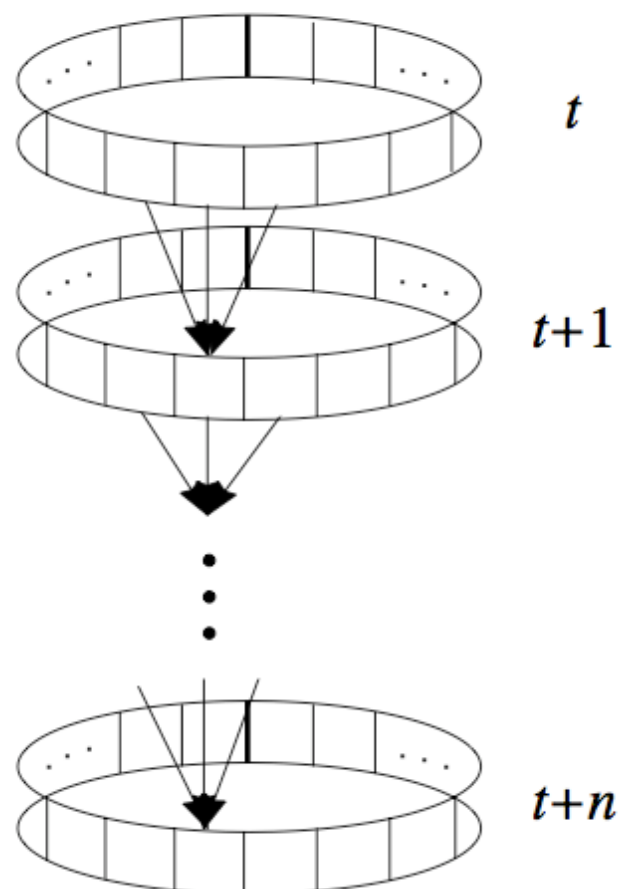
Dynamics in one dimension



boundary limit define a ring



evolution space



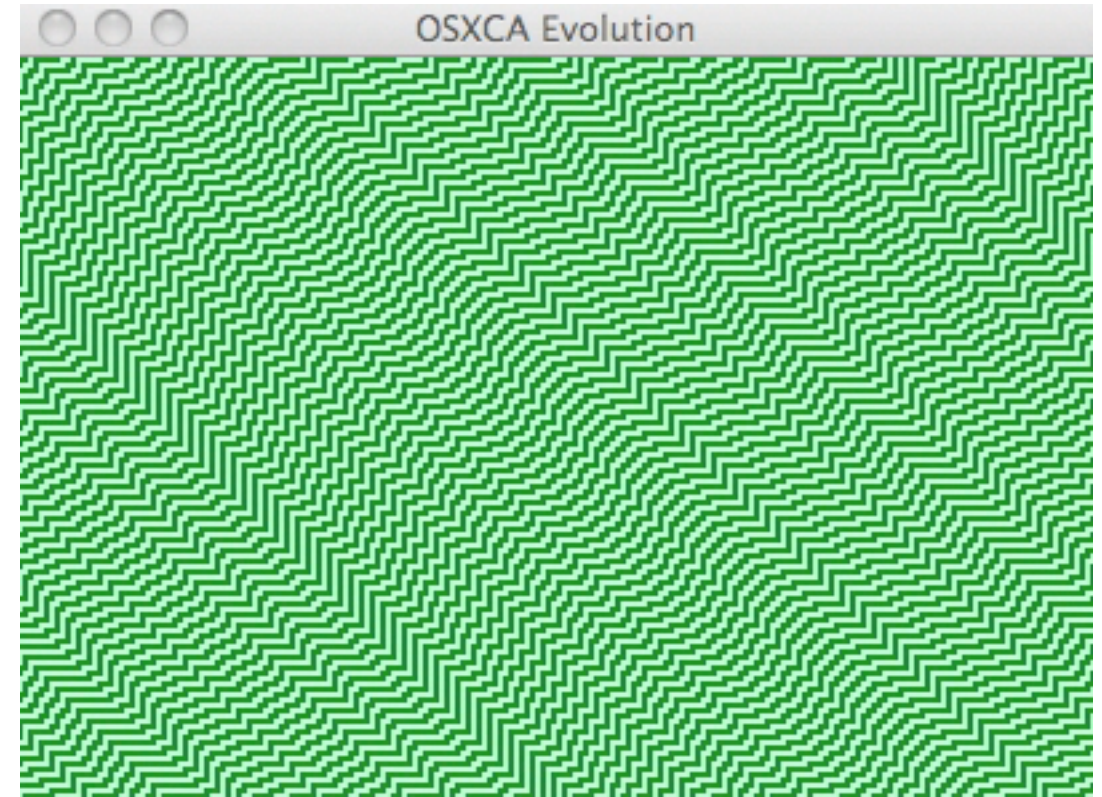
Elemental CA (ECA) is defined as follows:

- $\Sigma = \{0, 1\}$
- $\mu = (x_{+1}, x_0, x_{-1})$ such that $x \in \Sigma$
- $\phi : \Sigma^3 \rightarrow \Sigma$
- $\mu = \{c_0 \mid x \in \Sigma\}$ the initial condition is the first ring with $t = 0$

Cellular automata classes



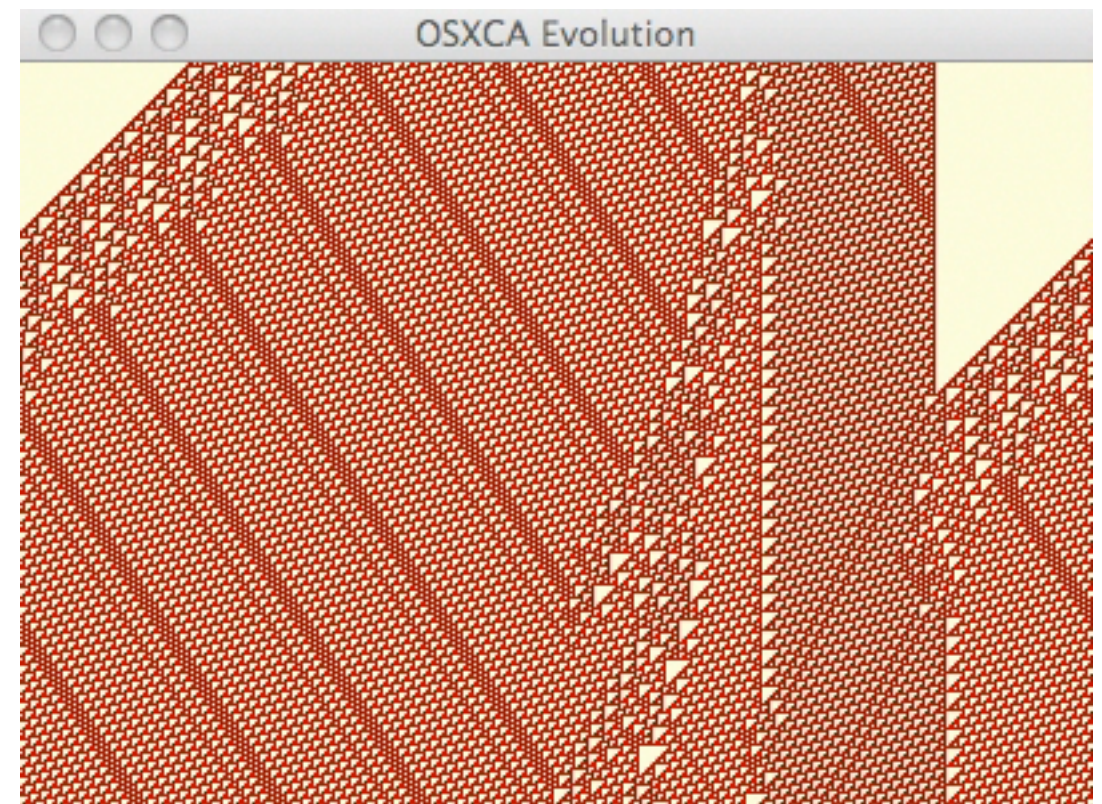
class I: uniform (rule 168)



class II: periodic (rule 15)



class III: chaotic (rule 30)



class IV: complex (rule 110)

Class I: evolve to *uniform* behaviour;

Class II: evolve to *periodic* behaviour;

Class III: evolve to *chaotic* behaviour;

Class IV: evolve to *complex* behaviour.

classification		
type	num.	rules
class I	8	0, 8, 32, 40, 128, 136, 160, 168.
class II	65	1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 19, 23, 24, 25, 26, 27, 28, 29, 33, 34, 35, 36, 37, 38, 42, 43, 44, 46, 50, 51, 56, 57, 58, 62, 72, 73, 74, 76, 77, 78, 94, 104, 108, 130, 132, 134, 138, 140, 142, 152, 154, 156, 162, 164, 170, 172, 178, 184, 200, 204, 232.
class III	11	18, 22, 30, 45, 60, 90, 105, 122, 126, 146, 150.
class IV	4	41, 54, 106, 110.

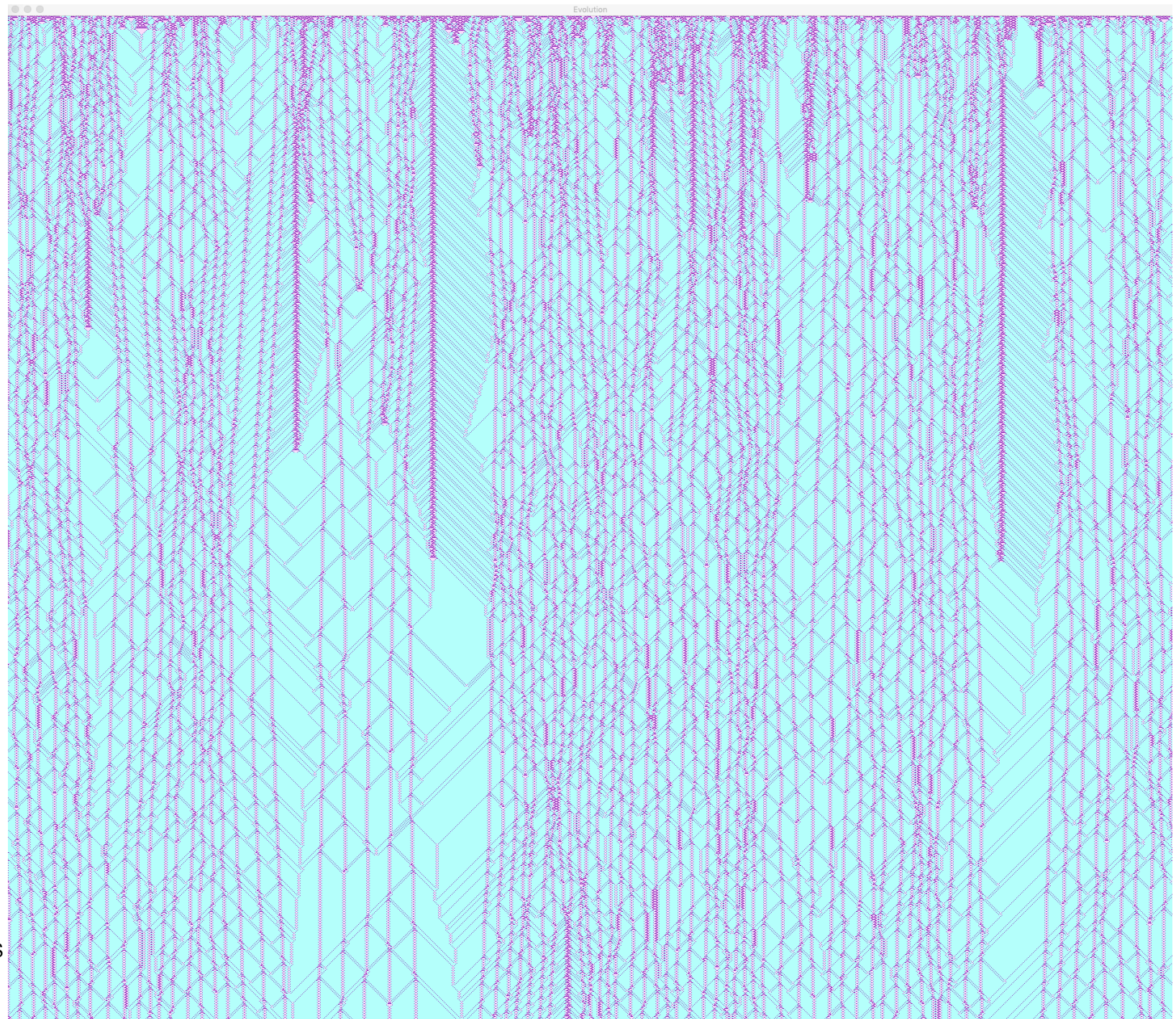
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Elementary cellular automaton rule 54

Some interesting points in rule 54:

- Artificial life
- Complex systems
- Logical computation
- Garden of Eden configurations
- Symmetric evolutions
- Guns emerge from random conditions

1900 cells x 1640 times



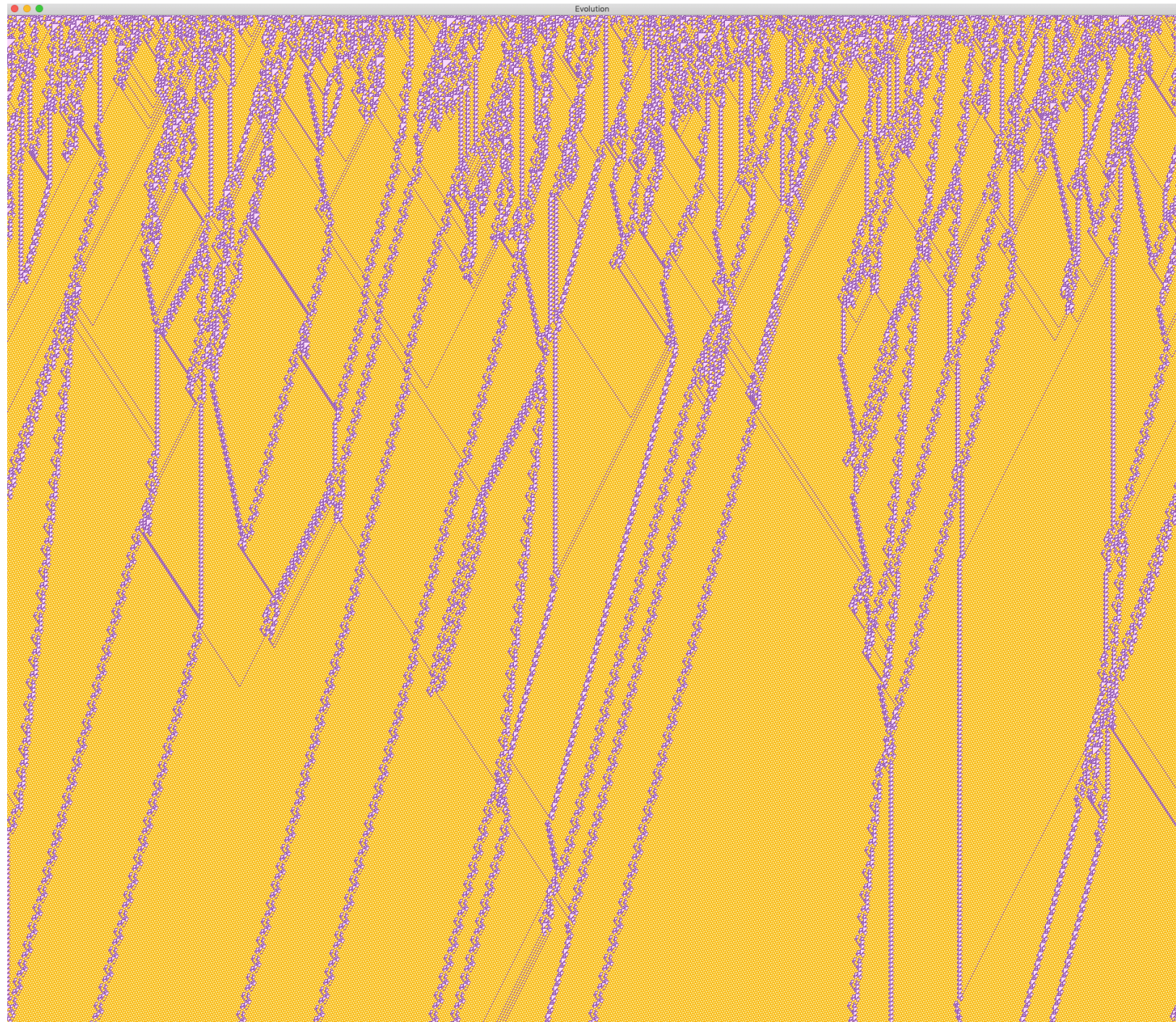
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- Guan, J. (2012) **Complex Dynamics of the Elementary Cellular Automaton Rule 54**, International Journal of Modern Physics C 23(7), 1250052.

Elementary cellular automaton rule 110

Some interesting points in rule 110:

- Artificial life
- Complex systems
- Universal computation
- Garden of Eden configurations
- Asymmetric evolutions
- Extendible gliders

1900 cells x 1640 times

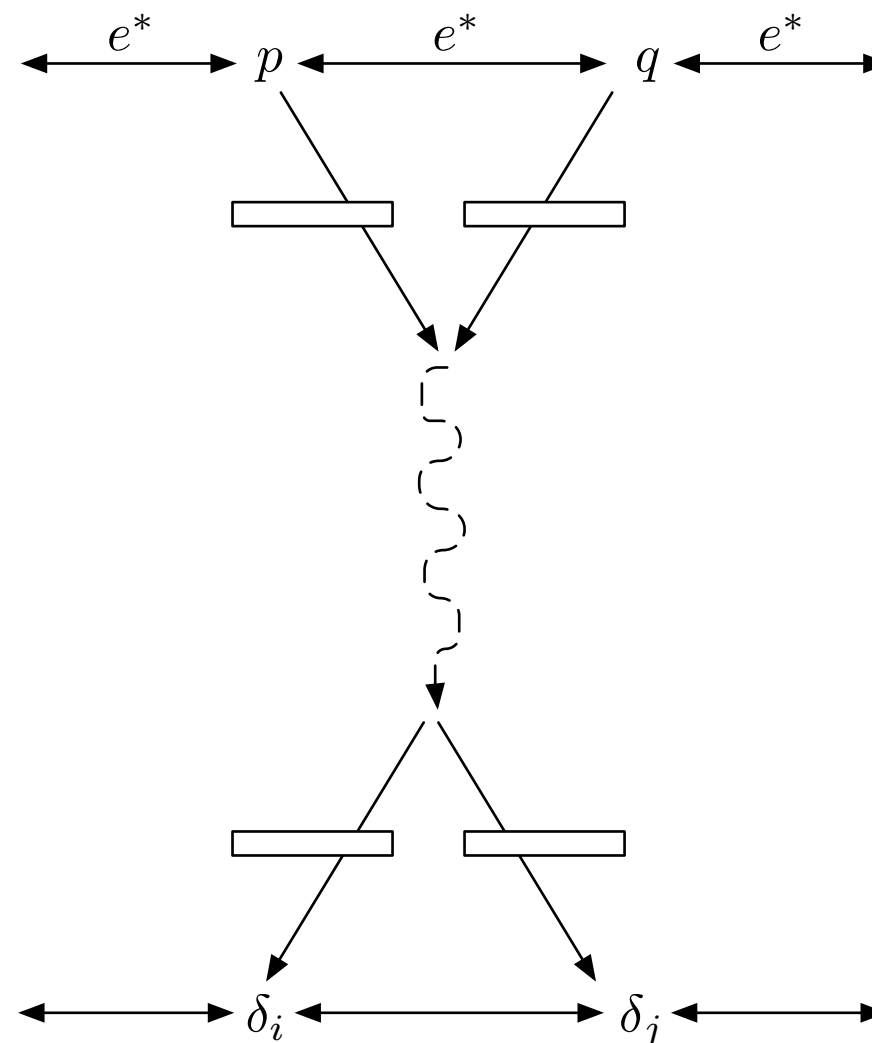


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Particles are strings

The formal languages theory provides a way to study sets of chains from a finite alphabet. The languages can be seen as inputs for some classes of machines or as the final result from a typesetter substitution system i.e., a generative grammar into the Chomsky's classification. This way, following a variation of a Feynman diagram hence we can represent collisions between particles in one-dimensional cellular automata as follows.

- p, q, δ – particles
- e – periodic background
- $\square$ – phase
- $f(p, q) \rightarrow \delta$



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- Hopcroft, J.E. & Ullman, J.D. (1987) **Introduction to Automata Theory Languages, and Computation**, Addison-Wesley Publishing Company.
- Collaborative Research Center SFB 676, **Particles, Strings, and the Early Universe**, Universität Hamburg , <http://wwwiexp.desy.de/sfb676/>

Particles are strings: Doug Lind starts this representation in ECA from 1986

Tables of Cellular Automaton Properties (1986)

Structures in rule 110.

The previous two pages show patterns produced by evolution according to rule 110, starting from a disordered initial configuration. The first picture shows all sites on a size 400 lattice. The second picture shows every other site in space and time on a size 800 lattice.

The configurations produced after many steps can be represented in terms of particle-like structures superimposed on a periodic background. The background is found to have spatial period 14 and temporal period 7, and corresponds to repetitions of the block **B** = 10011011111000. The configurations are then of the form ... **BBBBPBBB** ..., where the particles **P** that have been found so far are:

velocity	P
-6/12	1000110011101111111000
-2/4	11111000
-14/42	11100001110111111111000
-8/30	100110011000111111000
-4/15	00000
-4/36	111011111111000
-8/20	1111000011000
0/7	1111111000
0/7	100011000
0/7	10011011111111000
2/10	11101011000
2/10	1110100011011111000
2/3	111000

The “velocity” is written as (spatial period)/(temporal period).

One may speculate that the behaviour of rule 110 is sophisticated enough to support universal computation.

Table of particles by Doug Lind (*Mathematics Department, University of Washington, Seattle*).

Particles are strings and filters: James Hanson and James Crutchfield describing finite state machines in ECA from 1997

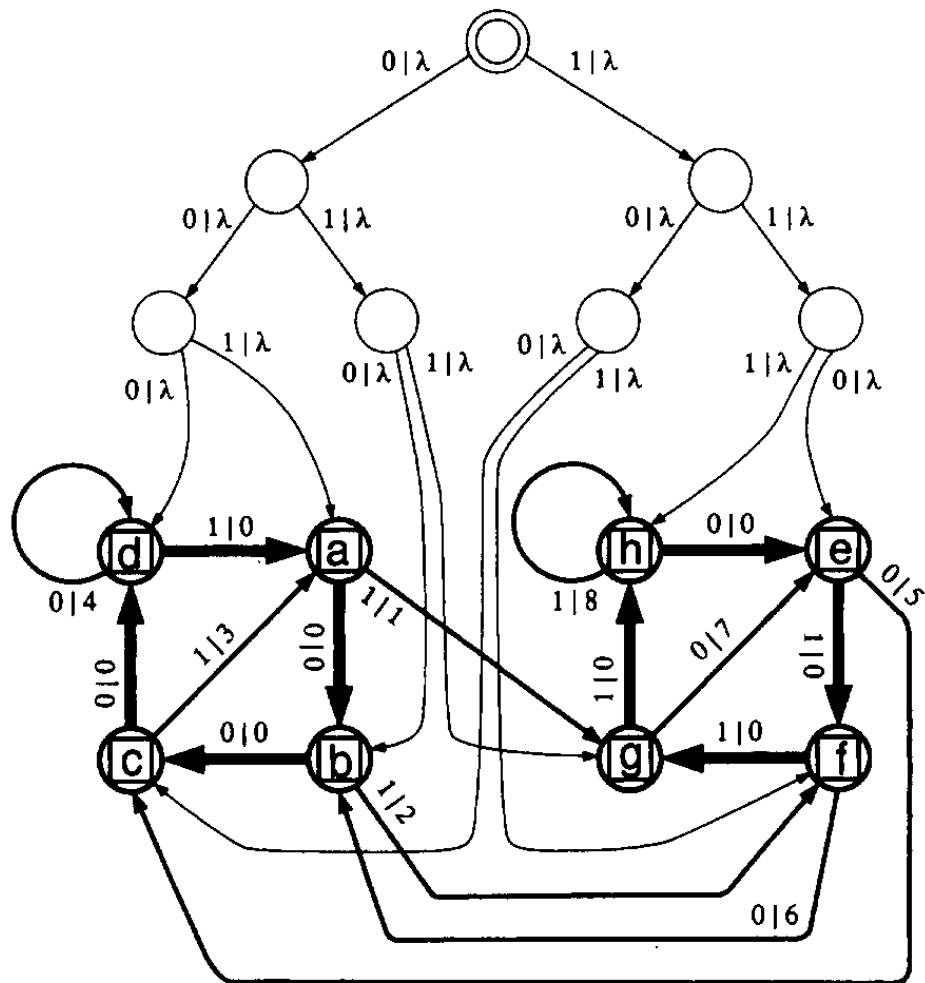


Fig. 5. ECA 54's domain filter T_{54}^0 , which maps sites in the domain to 0 and each defect to a unique output in $\{1, \dots, 8\}$. Labeled machine states correspond to the domain states of Fig. 3.

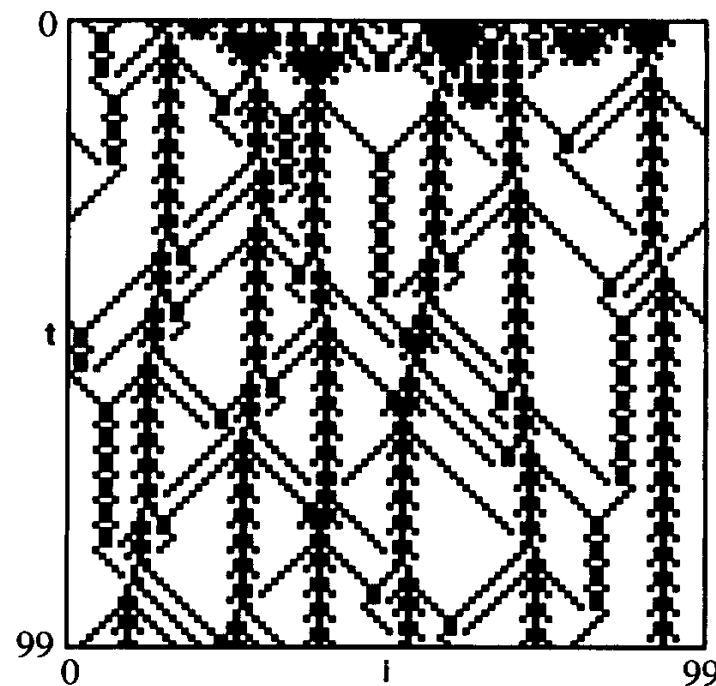


Fig. 6. Space-time data of Fig. 1, filtered with the domain transducer T_{54}^0 of Fig. 5. White cells correspond to sites participating in A_{54} ; black cells, to sites with values $s_t^i \in \{1, \dots, 8\}$.

Table 1
Fundamental interactions among ECA 54's particles

(a)	$\alpha + \gamma^- \rightarrow \gamma^- + \alpha + 2\gamma^+$
(b)	$\gamma^+ + \alpha \rightarrow 2\gamma^- + \alpha + \gamma^+$
(c)	$\beta + \gamma^- \rightarrow \gamma^+$
(d)	$\gamma^+ + \beta \rightarrow \gamma^-$
(e)	$\gamma^+ + \gamma^- \rightarrow \beta$
(f)	$\gamma^+ + \alpha + \gamma^- \rightarrow \gamma^- + \alpha + \gamma^+$
(g)	$\gamma^+ + \beta + \gamma^- \rightarrow \emptyset$

Particles are strings: Harold McIntosh established that the problem of rule 110 is a problem of tiles in 1998

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CHAPTER 1. OVERVIEW

1.4.4 ether crystallography

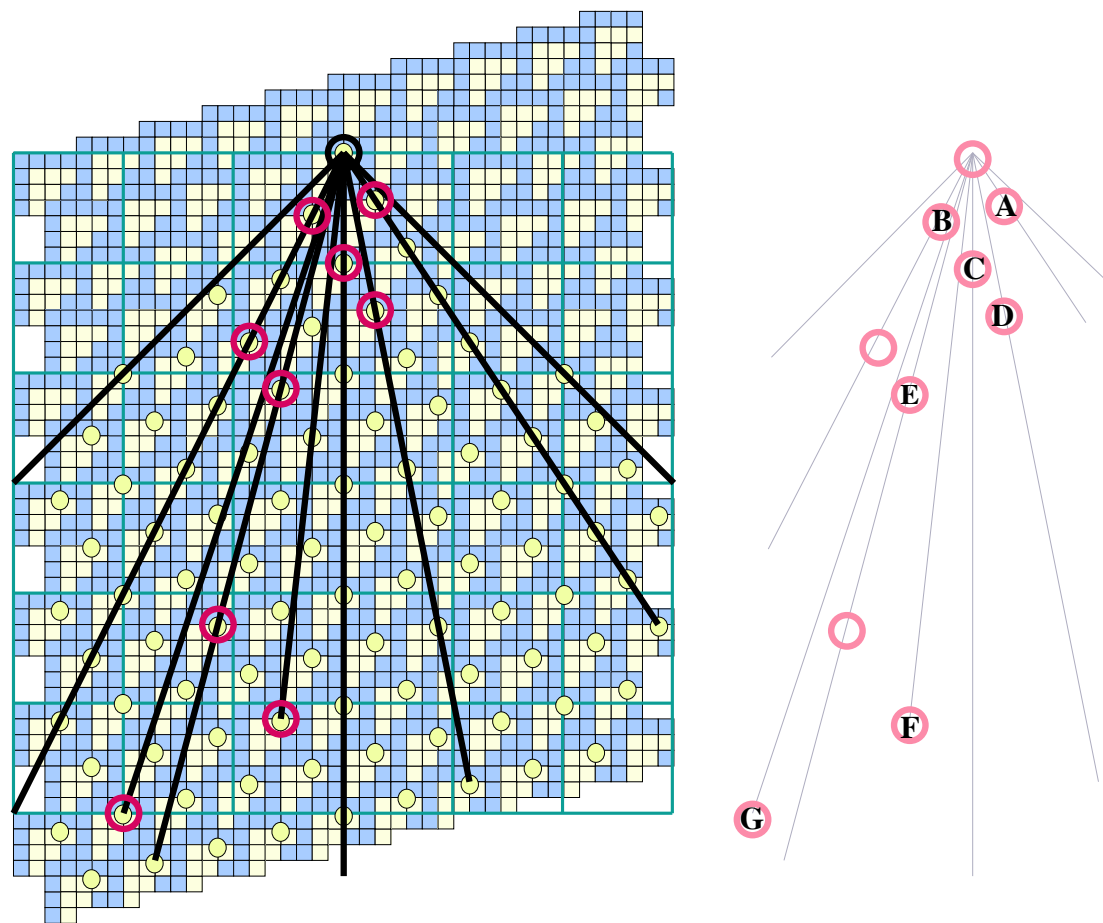
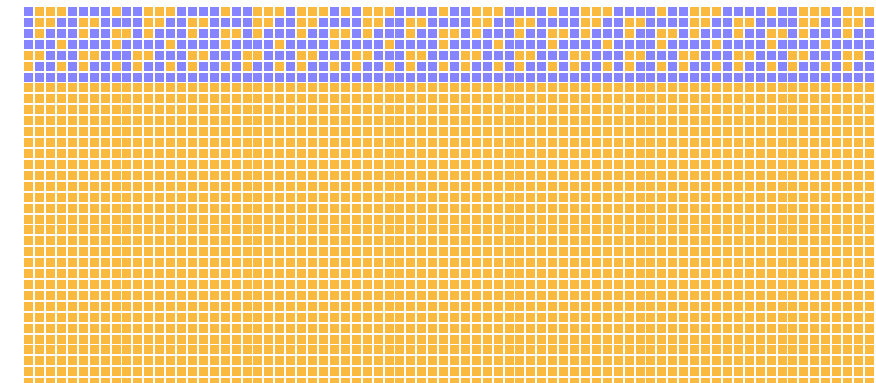


Figure 1.13: The locations of Cook's gliders relative to the ether lattice. The two barred gliders sit lower on the same velocity lines as the unbarred gliders. Small circles on the T3 mosaic show possible positions of compatible gliders, but they could be impossible, duplicates, or so far undiscovered.



evolving to 1 in 6 gen

110

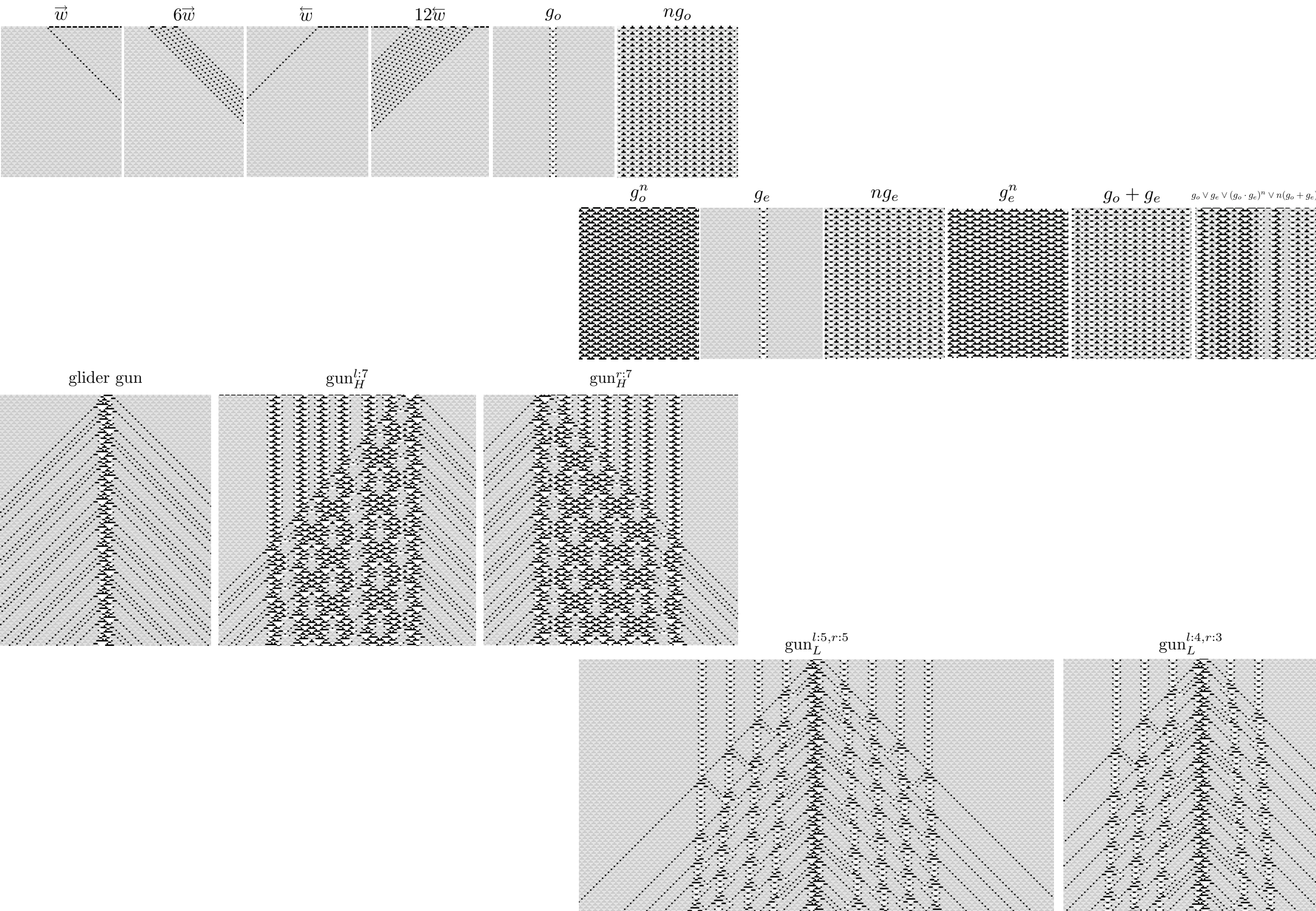
evolving to 1 in 6 generations

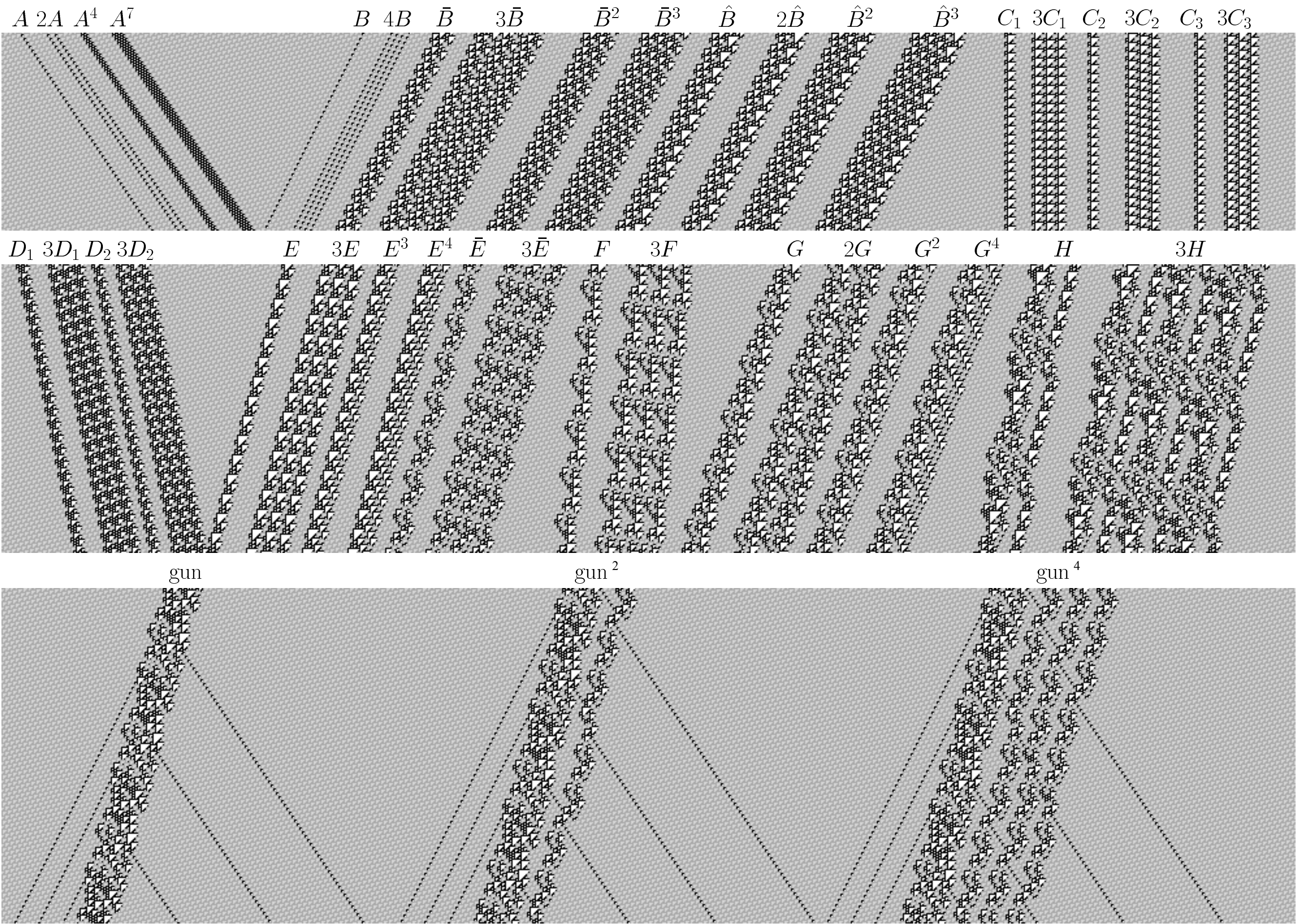
Rule 110

December 11, 1998

Figure 1.22: The de Bruijn diagram for evolution to the constant 1 after six generations.

Set of particles in rule 54 — <https://www.comunidad.escom.ipn.mx/genaro/Rule54.html>





Particles as strings from finite states machines

For an one-dimensional cellular automaton of order (k,r) , the *de Bruijn diagram* is defined as a directed graph with k^{2r} vertices and k^{2r+1} edges. The vertices are labeled with the elements of the alphabet of length $2r$. An edge is directed from vertex i to vertex j , if and only if, the $2r-1$ final symbols of i are the same that the $2r-1$ initial ones in j forming a neighbourhood of $2r+1$ states represented by $i \diamond j$. In this case, the edge connecting i to j is labeled with $\phi(i \diamond j)$.

The connection matrix M corresponding with the de Bruijn diagram is as follows:

$$M_{i,j} = \begin{cases} 1 & \text{if } j = ki, ki + 1, \dots, ki + k - 1 \pmod{k^{2r}} \\ 0 & \text{in other case} \end{cases}$$

Basins of attraction or cycle diagrams calculate attractors in a dynamical system, as was extensively studied by Andrew Wuensche in CA and random Boolean networks. Given a sequence of cells x_i we define a configuration c of the system. An evolution is represented by a sequence of configurations $c_0, c_1, c_2, \dots, c_{m-1}$ given by the global mapping,

$$\Phi : \Sigma^n \rightarrow \Sigma^n$$

and the global relation is given for the next function between configurations,

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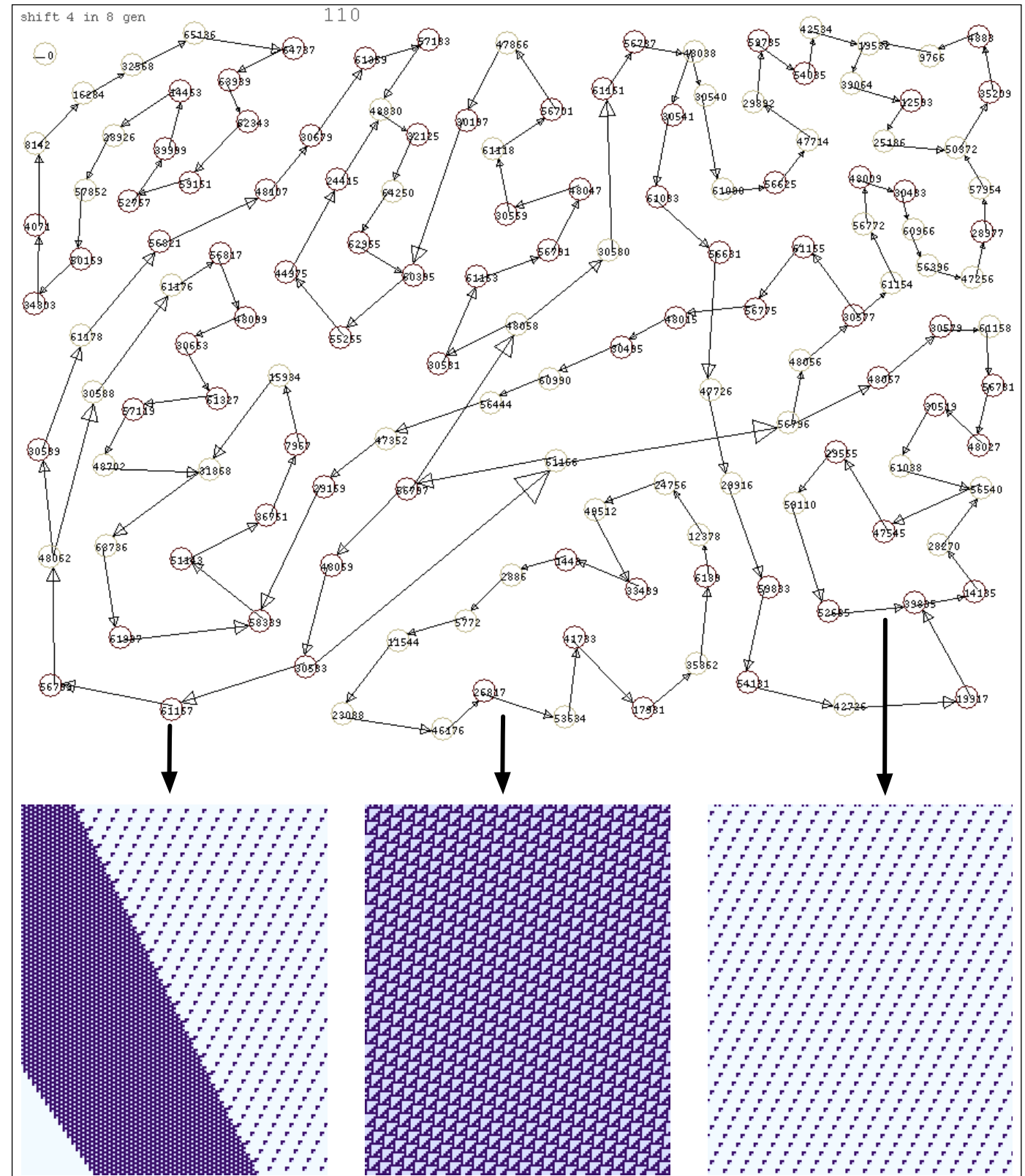
Particles as strings from finite states machines

De Bruijn diagram (displacement 4, period 8)
calculating non-stationary particles in rule 110.
The left evolution displays a *fuse pattern*
produced by two particles colliding and both
annihilated. The center evolution displays a
periodic pattern and the right evolution displays
particles with displacement to the left.

$$N = \{61166, 56799, 48059, 30583, 61167, 56703, 48062, 30589, 61178, 56821, 48107, 30679, 61369, 57183, 48830, 32125, 64250, 62965, 60395, 55255, 44975, 24415\}$$

$$((0111)^* + 11(01011111)^*)^*$$

It is an expression to codify A and B particles.

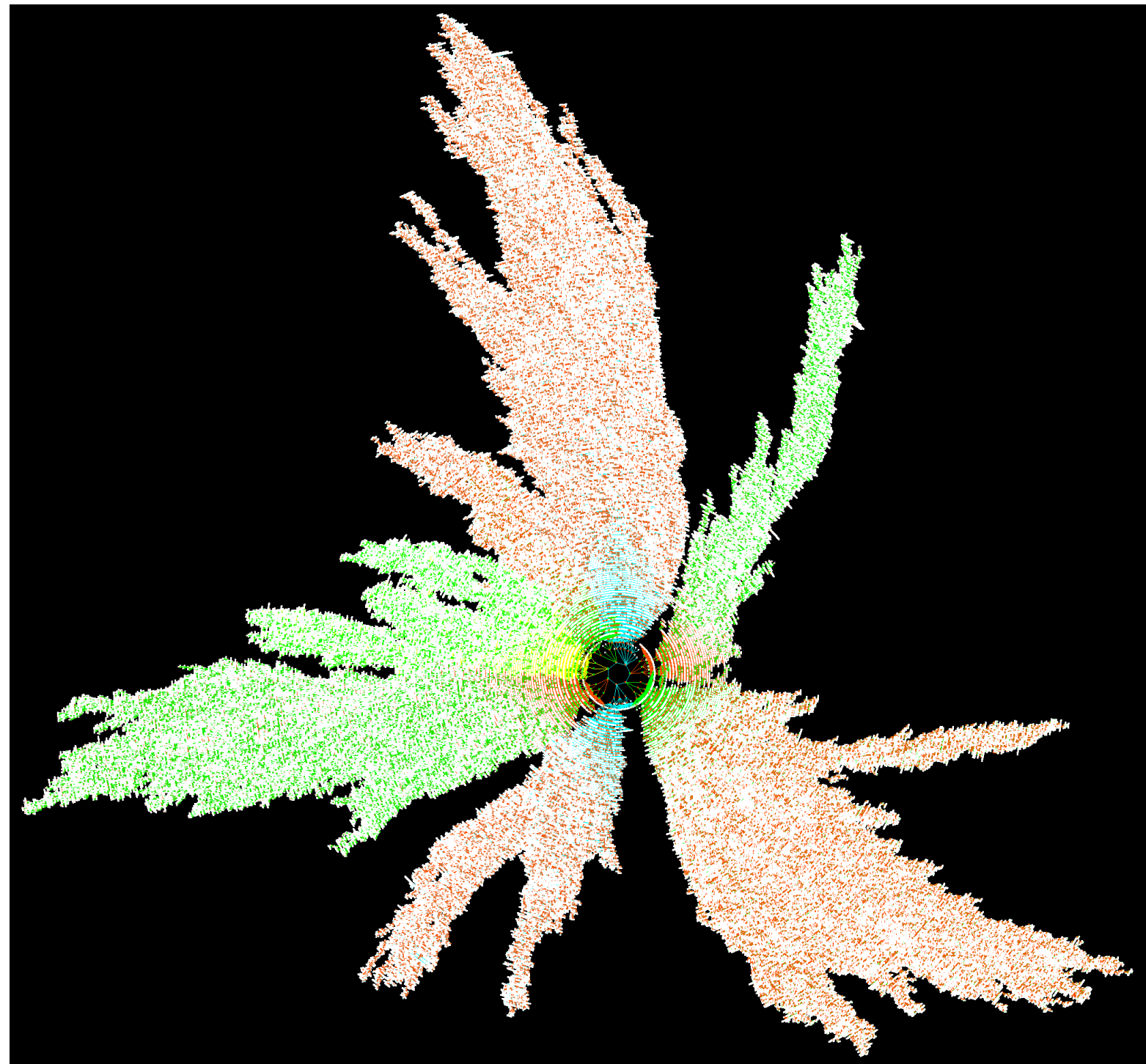
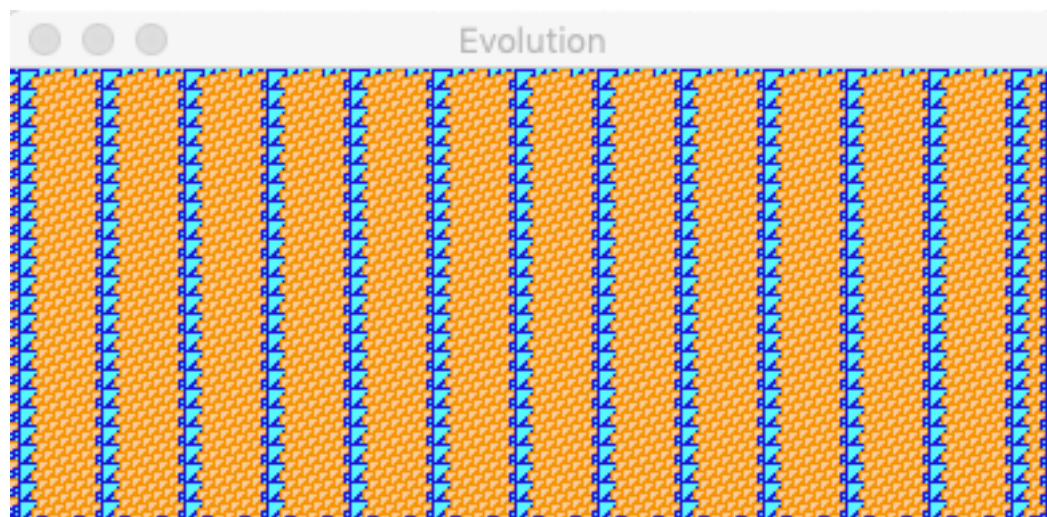


Particles as strings from finite states machines

Attractor length 31, period 7, with a mass of 5.487×10^7 configurations for rule 110. This attractor is in a field of 6,326 basins.

$w_0 = 0110111111110001001101111100010$
 $w_1 = 1111100000010011011111000100110$
 $w_2 = 1000100000110111110001001101111$
 $w_3 = 1001100001111100010011011111000$
 $w_4 = 1011100011000100110111110001001$
 $w_5 = 1110100111001101111100010011011$
 $w_6 = 0011101101011111000100110111110$

They are expressions to codify stationary (Cs) particles.



Particles as strings in rule 54

e	$\overrightarrow{w}$	$\overleftarrow{w}$
$e_1 = 1000$ $e_2 = 1110$	$\overrightarrow{w}(f_1) = e_1-10-e_2$ $\overrightarrow{w}(f_2) = e_2-00-e_1$ $2\overrightarrow{w}(f_1) = e_1-10111000-e_1$	$\overleftarrow{w}(f_1) = e_1-e_2$ $\overleftarrow{w}(f_2) = e_2-e_1$ $2\overleftarrow{w}(f_1) = e_1-11101000-e_1$ $2\overleftarrow{w}(f_2) = e_2-11101000-e_2$
	g_o	g_e
	$g_o(A, f_1) = e_1-100000-e_1$ $g_o(A, f_2) = e_2-111110-e_2$ $g_o(B, f_1) = e_1-10-e_1$ $g_o(B, f_2) = e_2-00-e_2$	$g_e(A, f_1) = e_1-1000000-e_1$ $g_e(A, f_2) = e_2-000-e_2$ $g_e(B, f_1) = e_1-100-e_1$ $g_e(B, f_2) = e_2-1111110-e_2$
	gun	
	$gun(A, f_1) = e_1-1111111100-e_1$ $gun(A, f_2) = e_2-1000000001-e_1$ $gun(B, f_1) = e_1-11100000010010-e_2$ $gun(C, f_1) = e_1-10001000011100-e_2$ $gun(C, f_2) = e_2-010001-e_1$ $gun(D, f_1) = e_1-1111010010-e_2$ $gun(D, f_2) = e_2-1000011111-e_1$ $gun(E, f_2) = e_2-11100100000010-e_2$ $gun(A2, f_1) = e_1-10001111000011-e_1$ $gun(A2, f_2) = e_2-10000100-e_1$ $gun(B2, f_1) = e_1-111001111110-e_2$ $gun(C2, f_1) = e_1-11000000-e_1$ $gun(C2, f_2) = e_2-10010000-e_2$ $gun(D2, f_1) = e_1-11111100-e_1$ $gun(D2, f_2) = e_2-100000011110-e_2$ $gun(E2, f_1) = e_1-111000010000-e_1$ $gun(A3, f_1) = e_1-10011100-e_2$ $gun(A3, f_2) = e_2-11110001-e_1$ $gun(B3, f_1) = e_1-100001010010-e_2$ $gun(B3, f_2) = e_2-01111111-e_1$ $gun(C3, f_1) = e_1-110000000010-e_2$ $gun(C3, f_2) = e_2-100100000011-e_1$ $gun(D3, f_1) = e_1-1111110000-e_1$ $gun(D3, f_2) = e_2-1000000100-e_2$ $gun(E3, f_1) = e_1-1110000111-e_1$ $gun(E3, f_2) = e_2-10001001000010-e_2$ $gun(A4, f_2) = e_2-1111110011-e_1$ $gun(B4, f_1) = e_1-10000001-e_1$ $gun(B4, f_2) = e_2-00010010-e_2$ $gun(C4, f_1) = e_1-10011111-e_1$ $gun(C4, f_2) = e_2-111100000010-e_2$ $gun(D4, f_1) = e_1-01000011-e_1$ $gun(D4, f_2) = e_2-011100-e_1$ $gun(E4, f_1) = e_1-110001-e_2$ $gun(E4, f_2) = e_2-1001010000-e_1$	

- Martínez, G.J., Adamatzky, A. & McIntosh, H.V. (2014) **Complete Characterization of Structure of Rule 54**, Complex Systems 23(3), 259-293.
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Particles as strings in rule 110

A glider

```
[111110] = A(f1_1), 6 cells, 1l-1r -> 1
[1111000110001010] = A(f2_1), 20 cells, 2l-3r -> 4
[11110001010101010] = A(f3_1), 20 cells, 3l-2r
A(f4_1) = A(f1_1)
```

B glider

```
[1111010] = B(f1_1), 8 cells, 1l-1r -> 2
[1111000] = B(f2_1), 8 cells, 2l-0r
[111100010110001010] = B(f3_1), 22 cells, 3l-3r -> 6
[111010] = B(f4_1), 8 cells, 0l-2r
```

B- glider

```
[111110001010111110010] = B-(A,f1_1), 22 cells, 1l-0r -> 0
[111100010011111100101101100010010] = B-(A,f2_1), 36 cells, 2l-3r -> 5
[111100010011110000010111000010010] = B-(A,f3_1), 36 cells, 3l-2r
[1111100001110010010] = B-(A,f4_1), 22 cells, 0l-1r
[11110000100010101010] = B-(B,f1_1), 22 cells, 1l-0r -> 0
[111100010001100111011111000010010] = B-(B,f2_1), 36 cells, 2l-3r -> 5
[111100010011001101110000010010] = B-(B,f3_1), 36 cells, 3l-2r
[1101010111101000010] = B-(B,f4_1), 22 cells, 0l-1r
[1111011111000011000] = B-(C,f1_1), 22 cells, 1l-0r -> 0
[11110001100001000110011000100010] = B-(C,f2_1), 36 cells, 2l-3r -> 5
[1111000100101000010010111011001010] = B-(C,f3_1), 36 cells, 3l-2r
[1111001110110001010] = B-(C,f4_1), 22 cells, 0l-1r
```

B^ glider

```
[1111000101011110001001111100010010] = B^(A,f1_1), 39 cells, 1l-3r -> 4
[111100010011111100101111000010010] = B^(A,f2_1), 39 cells, 2l-2r
[11110001001100000101110111000010] = B^(A,f3_1), 39 cells, 3l-1r
[1111100001110010000] = B^(A,f4_1), 22 cells, 0l-0r -> 0
[11110001001100101010100100010010] = B^(B,f1_1), 39 cells, 1l-3r -> 4
[111100001000110011111101110010] = B^(B,f2_1), 39 cells, 2l-2r
[11110001001101011100000000110] = B^(B,f3_1), 39 cells, 3l-1r
[1101010111101000000010] = B^(B,f4_1), 22 cells, 0l-0r -> 0
[1111011111000011000000111100010010] = B^(C,f1_1), 39 cells, 1l-3r -> 4
[1111000110000100010100000100010010] = B^(C,f2_1), 39 cells, 2l-2r
[1111000101010001011110000110010] = B^(C,f3_1), 39 cells, 3l-1r
[11110011100100010001010] = B^(C,f4_1), 22 cells, 0l-0r -> 0
```

C1 glider

```
[11110000] = C1(A,f1_1), 9 cells, 1l-0r -> 0
[11110001000100010010] = C1(A,f2_1), 23 cells, 2l-3r -> 4
[111100010001000110010] = C1(A,f3_1), 23 cells, 3l-2r
[1101010] = C1(A,f4_1), 9 cells, 0l-1r
[1111011111100010010] = C1(B,f1_1), 23 cells, 1l-3r -> 4
[11110001100000010010] = C1(B,f2_1), 23 cells, 2l-2r
[11110001010100000010] = C1(B,f3_1), 23 cells, 3l-1r
C1(B,f4_1) = C1(B,f1_1)
```

C2 glider

```
[111100000010010] = C2(A,f1_1), 17 cells, 1l-2r -> 2-3
[111100010000010] = C2(A,f2_1), 17 cells, 2l-1r
[11110001010000] = C2(A,f3_1), 17 cells, 3l-0r
[111000100010010] = C2(A,f4_1), 17 cells, 0l-3r
[11110001110010] = C2(B,f1_1), 17 cells, 1l-2r -> 2-3
[11110001101010] = C2(B,f2_1), 17 cells, 2l-1r
[111100010010101111100010010] = C2(B,f3_1), 31 cells, 3l-3r -> 6
C2(B,f4_1) = C2(B,f1_1)
```

C3 glider

```
[111101010] = C3(A,f1_1), 17 cells, 1l-1r -> 1
[111100011111100010010] = C3(A,f2_1), 31 cells, 2l-3r -> 4-5
[111100010100000010] = C3(A,f3_1), 31 cells, 3l-2r
[110000010] = C3(A,f4_1), 17 cells, 0l-1r
[1111010000] = C3(B,f1_1), 17 cells, 1l-0r -> 1
[111100011000100010010] = C3(B,f2_1), 31 cells, 2l-3r -> 4-5
[1111000100100100110010] = C3(B,f3_1), 31 cells, 3l-2r
C3(B,f4_1) = C3(B,f1_1)
```

D1 glider

```
[1111000010] = D1(A,f1_1), 11 cells, 1l-1r -> 0
[1111000100011100010010] = D1(A,f2_1), 25 cells, 2l-3r -> 4
[111100010011001001010] = D1(A,f3_1), 25 cells, 3l-2r
[110101010] = D1(A,f4_1), 11 cells, 0l-1r
[111101101] = D1(B,f1_1), 25 cells, 1l-3r -> 4
[11110001101100000100] = D1(B,f2_1), 25 cells, 2l-2r
[1111000100101101000010] = D1(B,f3_1), 25 cells, 3l-1r
D1(B,f4_1) = D1(C,f1_1)
[111101100] = D1(C,f1_1), 11 cells, 1l-0r -> 0
[11110001010100010010] = D1(C,f2_1), 25 cells, 2l-3r -> 4
[11110001001011110010] = D1(C,f3_1), 25 cells, 3l-2r
D1(C,f4_1) = D1(A,f1_1)
```

D2 glider

```
[111101010100010010] = D2(A,f1_1), 19 cells, 1l-3r -> 3-3-2
[1111000111110010] = D2(A,f2_1), 19 cells, 2l-2r
[1111000100110000010] = D2(A,f3_1), 19 cells, 3l-1r
[1111000111000010] = D2(B,f1_1), 19 cells, 0l-3r
[11110100100100010] = D2(B,f2_1), 19 cells, 1l-2r -> 3-3-2
[111100011011010] = D2(B,f2_1), 19 cells, 2l-1r
[1111000101010110111100010010] = D2(B,f3_1), 33 cells, 3l-3r -> 6-6-6
D2(B,f4_1) = D2(C,f1_1)
[11110111000010010] = D2(C,f1_1), 19 cells, 1l-2r -> 3-3-2
[1111000110100010] = D2(C,f2_1), 19 cells, 2l-1r
[1111000101011100] = D2(C,f3_1), 19 cells, 3l-0r
D2(C,f4_1) = D2(A,f1_1)
```

E glider

```
[11110000000010010] = E(A,f1_1), 19 cells, 1l-2r -> 3-3-3-2
[11110001000000010] = E(A,f2_1), 19 cells, 2l-1r
[1111000110000000] = E(A,f3_1), 19 cells, 3l-0r
[11100001000010010] = E(A,f4_1), 19 cells, 0l-3r
[1111000100010] = E(B,f1_1), 19 cells, 1l-2r
[1111000110001010] = E(B,f2_1), 19 cells, 2l-1r
```

```
[11110001001010011111100010010] = E(B,f3_1), 33 cells, 3l-3r -> 7-7-6-6
E(B,f4_1) = E(C,f1_1)
[1111010000010010] = E(C,f1_1), 19 cells, 1l-2r -> 3-3-3-2
[1111000111000010] = E(C,f2_1), 19 cells, 2l-1r
[111100010010000] = E(C,f3_1), 19 cells, 3l-0r
[1101001000100010] = E(C,f4_1), 19 cells, 0l-3r
[111101110110010] = E(D,f1_1), 19 cells, 1l-2r -> 3-3-3-2
[1111000110011010] = E(D,f2_1), 19 cells, 2l-1r
[11110001001010] = E(D,f3_1), 19 cells, 3l-0r
[1111111100010010] = E(D,f4_1), 19 cells, 0l-3r
```

E- glider

```
[1111000010001111010] = E-(A,f1_1), 21 cells, 1l-1r -> 2
[1111000100010001000] = E-(A,f2_1), 21 cells, 2l-0r -> 2
[111100010010011011011000100010] = E-(A,f3_1), 35 cells, 3l-3r -> 6-6-6-6-6-6-5-5
[110101011010110010] = E-(A,f4_1), 21 cells, 0l-2r
[11110111101010] = E-(B,f1_1), 21 cells, 1l-1r -> 2
[111100011000011000] = E-(B,f2_1), 21 cells, 2l-0r
[11110001001000010] = E-(B,f3_1), 35 cells, 3l-3r
[11110011110110010] = E-(B,f4_1), 21 cells, 0l-2r
[111100010100011010] = E-(C,f1_1), 21 cells, 1l-1r -> 2
[11110001001110010] = E-(C,f2_1), 21 cells, 2l-0r
[11110001010100010111000010] = E-(C,f3_1), 35 cells, 3l-3r
[1111000111000010] = E-(C,f4_1), 21 cells, 0l-2r
[111100001001000100] = E-(D,f1_1), 21 cells, 1l-1r -> 2
[1111000100010010010] = E-(D,f2_1), 21 cells, 2l-0r
[111100010011011010] = E-(D,f3_1), 35 cells, 3l-3r
[110101001000010010] = E-(D,f4_1), 21 cells, 0l-2r
[11110110101100010] = E-(E,f1_1), 21 cells, 1l-1r -> 2
[11110001101110100] = E-(E,f2_1), 21 cells, 2l-0r
[111100010010010000010010010] = E-(E,f3_1), 35 cells, 3l-3r
E-(E,f4_1) = E-(F,f1_1)
[11110100101110010] = E-(F,f1_1), 21 cells, 1l-1r -> 2
[111100011110010] = E-(F,f2_1), 21 cells, 2l-0r
[1111000100101100001011100010010] = E-(F,f3_1), 35 cells, 3l-3r
E-(F,f4_1) = E-(G,f1_1)
[111101001011100010010] = E-(G,f1_1), 21 cells, 1l-1r -> 2
[11110001001010010] = E-(G,f2_1), 21 cells, 2l-0r
[1111000100101101111100010010] = E-(G,f3_1), 35 cells, 3l-3r
[11110011000000100] = E-(G,f4_1), 21 cells, 0l-2r
[111100010101000010] = E-(H,f1_1), 21 cells, 1l-1r -> 2
[11110000011111000] = E-(H,f2_1), 21 cells, 2l-0r
[11110001001010000010010010] = E-(H,f3_1), 35 cells, 3l-3r
[11110000101100010] = E-(H,f4_1), 21 cells, 0l-2r
```

F glider

```
[11110001010] = F(A,f1_1), 15 cells, 1l-1r -> destroy
[111100010111100010010] = F(A,f2_1), 29 cells, 2l-3r -> 3-4-4-5-5-5-4-4-4-4
[1111000100101000000010010] = F(A,f3_1), 29 cells, 3l-2r
[1111100000010] = F(A,f4_1), 15 cells, 0l-1r -> destroy
[11110000100000] = F(B,f1_1), 15 cells, 1l-0r -> destroy
[1111000100010000100010010] = F(B,f2_1), 29 cells, 2l-3r -> 3-4-4-5-5-5-4-4-4-4
[1111000100100011000110010] = F(B,f3_1), 29 cells, 3l-2r
[11010100010] = F(B,f4_1), 15 cells, 0l-1r
[111101111010111100010010] = F(C,f1_1), 29 cells, 1l-3r -> 3-4-4-5-5-5-4-4-4-4
[1111000110000100000010010] = F(C,f2_1), 29 cells, 2l-2r
[1111000100101000010000010] = F(C,f3_1), 29 cells, 3l-1r
[11110111100] = F(C,f4_1), 15 cells, 0l-0r -> destroy
[11110001001000100010010] = F(D,f1_1), 29 cells, 1l-3r -> 3-4-4-5-5-5-4-4-4-4
[111100010011100010110010] = F(D,f2_1), 29 cells, 2l-2r
[11110001001010000101110] = F(D,f3_1), 29 cells, 3l-1r
[11111011100] = F(D,f4_1), 15 cells, 0l-0r -> destroy
[1111000010100100100010010] = F(E,f1_1), 29 cells, 1l-3r -> 3-4-4-5-5-5-4-4-4-4
[11110001000111010110010] = F(E,f2_1), 29 cells, 2l-2r
[11110001000] = F(E,f4_1), 15 cells, 0l-0r -> destroy
[1111011010110001000010010] = F(F,f1_1), 29 cells, 1l-3r -> 3-4-4-5-5-5-4-4-4-4
[111100011011101001010010] = F(F,f2_1), 29 cells, 2l-2r
[111100010010110001010010] = F(F,f3_1), 29 cells, 3l-1r
F(F,f4_1) = F(G,f1_1)
[1111010010101111100010010] = F(G,f1_1), 29 cells, 1l-3r -> 3-4-4-5-5-5-4-4-4-4
[111100011011000100010] = F(G,f2_1), 29 cells, 2l-2r
[11110001001001100110001000010] = F(G,f3_1), 29 cells, 3l-1r
F(G,f4_1) = F(H,f1_1)
[11110100100000] = F(H,f1_1), 15 cells, 1l-0r -> destroy
[1111000110110001000010010] = F(H,f2_1), 29 cells, 2l-3r -> 3-4-4-5-5-5-4-4-4-4
[111100010010101000110010] = F(H,f3_1), 29 cells, 3l-1r
F(H,f4_1) = F(A2,f1_1)
[111101101010] = F(A2,f1_1), 15 cells, 1l-1r -> destroy
[1111000110111110000010010] = F(A2,f2_1), 29 cells, 2l-3r -> 3-4-4-5-5-5-4-4-4-4
[11110001001011000000010010] = F(A2,f3_1), 29 cells, 3l-2r
F(A2,f4_1) = F(B2,f1_1)
[11110100000] = F(B2,f1_1), 15 cells, 1l-1r -> destroy
[1111000110000] = F(B2,f2_1), 15 cells, 2l-0r
[1111000100101000100010010] = F(B2,f3_1), 29 cells, 3l-3r -> 3-4-4-5-5-5-4-4-4-4
[1111001110010] = F(B2,f4_1), 15 cells, 0l-2r
```

G glider

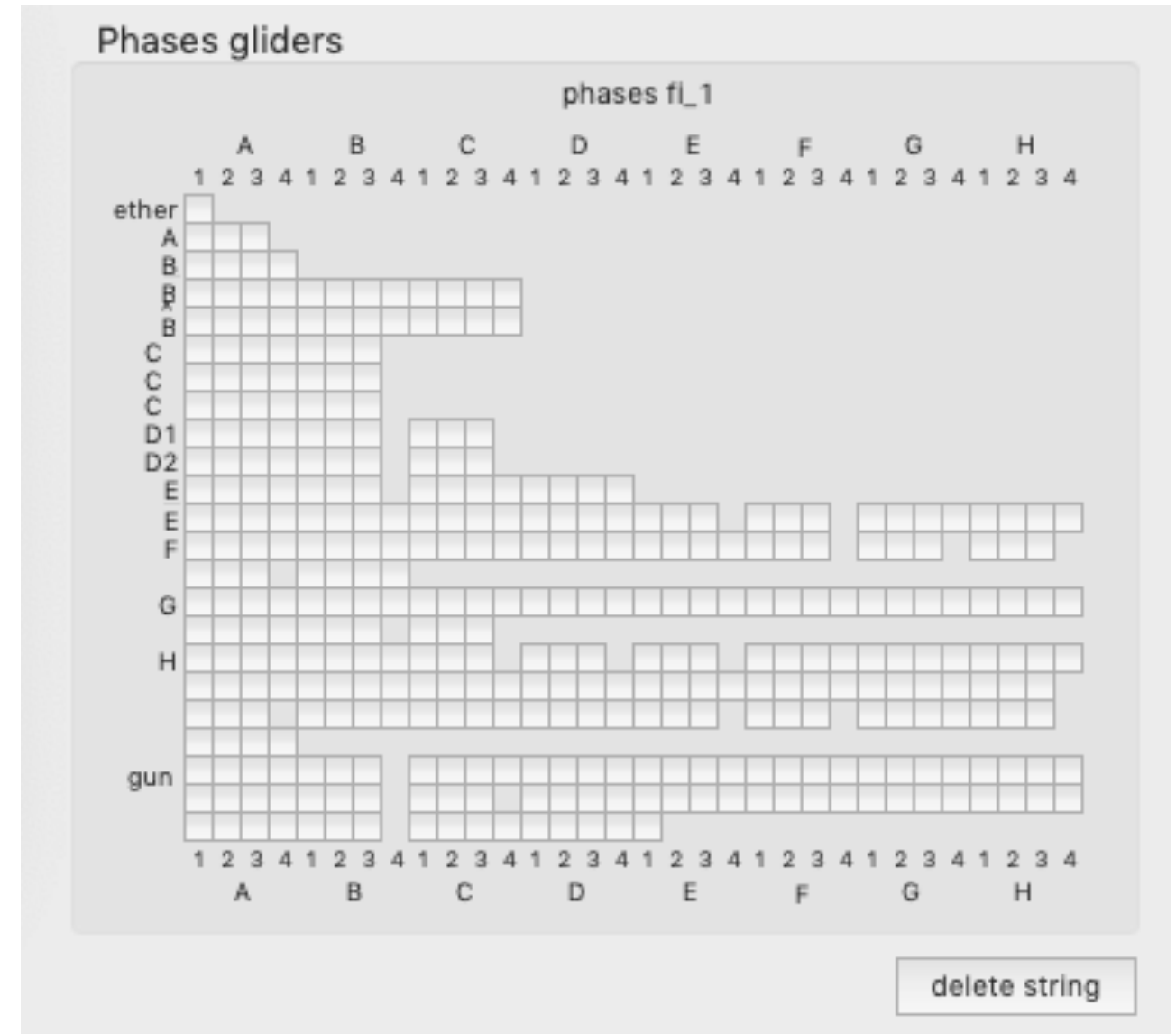
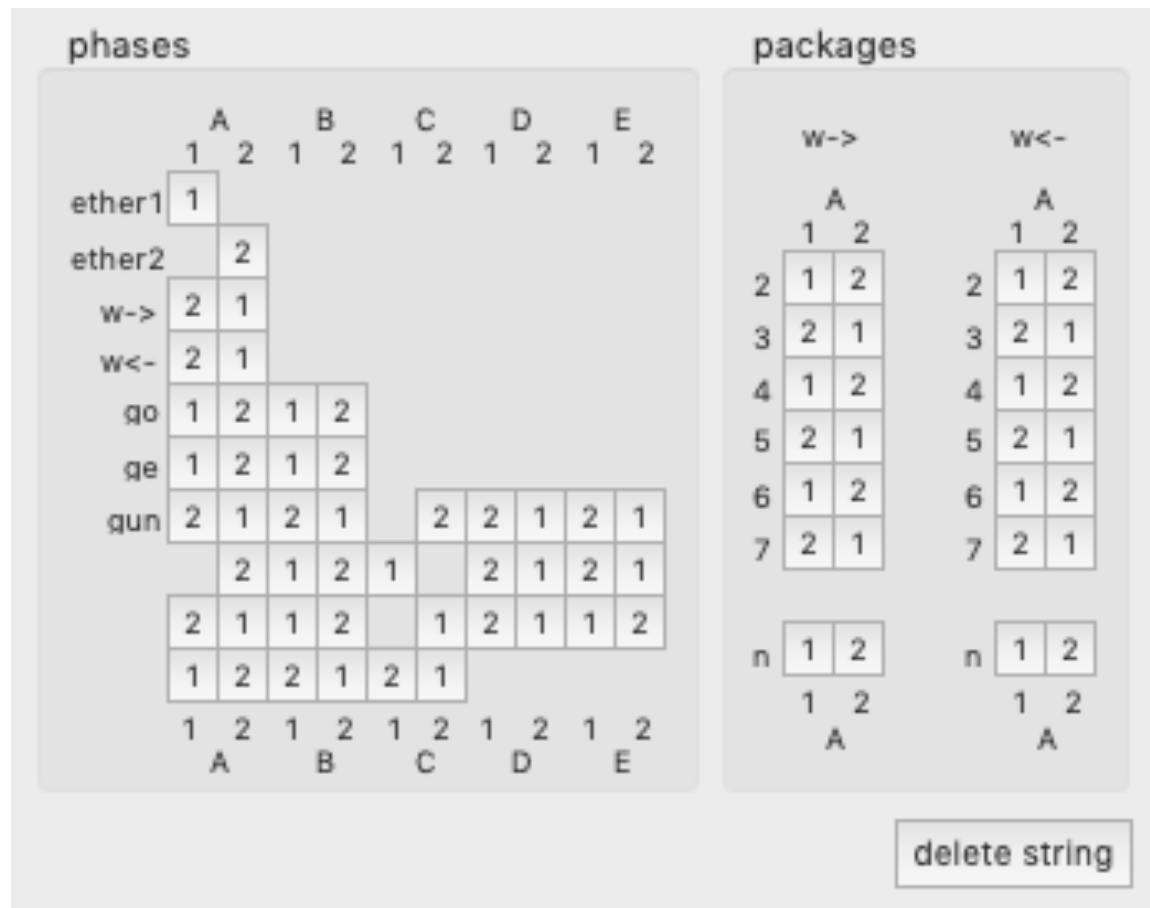
```
[111101000111000110010] = G(A,f1_1), 24 cells, 1l-1r -> destroy
[1111000110100001010] = G(A,f2_1), 24 cells, 2l-0r -> destroy
[111100010010111000111111000010010] = G(A,f3_1), 38 cells, 3l-3r -> 4-4-4-5-6-5-5-5-6-5-5-5
[111100101000000010010] = G(A,f4_1), 24 cells, 0l-2r
[11110001011100000010] = G(B,f1_1), 24 cells, 1l-1r -> destroy
[11110001011000100000] = G(B,f2_1), 24 cells, 2l-0r
[111100010010100000111111000010010] = G(B,f3_1), 38 cells, 3l-3r -> 4-4-4-5-6-5-5-5-6-5-5-5
[11110101010001100010] = G(B,f4_1), 24 cells, 0l-2r
[1111000011111010010] = G(C,f1_1), 24 cells, 1l-1r -> destroy
[111100010001000011011111000010010] = G(C,f2_1), 38 cells, 2l-3r -> 4-4-4-5-6-5-5-5-6-5-5-5
[1111000100110001011100000010010] = G(C,f3_1), 38 cells, 3l-2r
[1111000100110001011100000010010] = G(C,f4_1), 24 cells, 0l-1r
[110101000111101000010] = G(C,f4_1), 24 cells, 0l-1r
[11110111101001000] = G(D,f1_1), 24 cells, 1l-0r -> destroy
[11110001000011000101000100010] = G(D,f2_1), 38 cells, 2l-3r -> 4-4-4-5-6-5-5-5-6-5-5-5
[11110001001010000100011101010010] = G(D,f3_1), 38 cells, 3l-2r
[11110011101110001010] = G(D,f4_1), 24 cells, 0l-1r
[111100010101100010010] = G(E,f1_1), 24 cells, 1l-0r -> destroy
[111100010011110101011100000010] = G(E,f2_1), 38 cells, 2l-3r -> 4-4-4-5-6-5-5-5-6-5-5-5
[11110001001000001110100010010] = G(E,f3_1), 38 cells, 3l-2r
```

```
[1110101110001000110010] = G(F,f4_1), 24 cells, 0l-1r
[1111011110100101010] = G(G,f1_1), 24 cells, 1l-0r -> destroy
[11110001100001010111111100010010] = G(G,f2_1), 38 cells, 2l-3r -> 4-4-4-5-6-5-5-5-6-5-5-5
[111100010010100001011000000010010] = G(G,f3_1), 38 cells, 3l-2r
[111100011101000000010] = G(G,f4_1), 24 cells, 0l-1r
[11110001001000100000] = G(H,f1_1), 24 cells, 1l-0r -> destroy
[111100001111001010000010000100010] = G(H,f2_1), 38 cells, 2l-3r -> 4-4-4-5-6-5-5-5-6-5-5-5
[11110001001010000101110000110010] = G(H,f3_1), 38 cells, 3l-2r
[1111100111000100001010] = G(H,f4_1), 24 cells, 0l-1r
[11110000010010010001111100010010] = G(A2,f1_1), 38 cells, 1l-1r -> 4-4-4-5-6-5-5-5-6-5-5-5
[111100010001111010101100000010010] = G(A2,f2_1), 38 cells, 2l-2r
[111100010011000100010110111000010] = G(A2,f3_1), 38 cells, 3l-1r
[110101001000011001000] = G(A2,f4_1), 24 cells, 0l-0r -> destroy
[111101010101000101010001000010010] = G(B2,f1_1), 38 cells, 1l-3r -> 4-4-4-5-6-5-5-5-6-5-5-5
[1111000110101110001111010110010] = G(B2,f2_1), 38 cells, 2l-2r
[111100010010110000101100000010010] = G(B2,f3_1), 38 cells, 3l-1r
G(B2,f4_1) = G(C2,f1_1)
[111100010101100000010] = G(C2,f1_1), 24 cells, 1l-0r -> destroy
[11110001101110010000111000010010] = G(C2,f2_1), 38 cells, 2l-3r -> 4-4-4-5-6-5-5-5-6-5-5-5
[1111000100101100001100001000010010] = G(C2,f3_1), 38 cells, 3l-2r
G(C2,f4_1) = G(A,f1_1)
```

H glider

```
[1111000101000000001111000100101001111100010010] = H(A,f1_1), 53 cells, 1l-3r -> 3-4-5-5-5-4-4-5-4-4-4-4-4-4-5-6-5-5-5-5-4-4-4-4
[11110001001110000000010000001001010111000000010010] = H(A,f2_1), 53 cells, 2l-2r
[11110001001010001000000010010101110001110000010] = H(A,f3_1), 53 cells, 3l-1r
[1111001000001010111100001001000] = H(A,f4_1), 39 cells, 0l-0r -> destroy
[11110000010100001111100000010101001000100010] = H(B,f1_1), 53 cells, 1l-3r -> 3-4-5-5-4-4-5-4-4-4-4-4-5-6-5-5-5-5-4-4-4-4
[111100010001101000010000000100101110110110010] = H(B,f2_1), 53 cells, 2l-2r
[11110001001001000110000010110000101100011001010] = H(B,f3_1), 53 cells, 3l-1r
[1110101010101000011100001001010] = H(B,f4_1), 39 cells, 0l-0r -> destroy
[111101101111111000100010010101111100010010] = H(C,f1_1), 53 cells, 1l-3r -> 3-4-5-5-4-4-5-4-4-4-4-4-5-6-5-5-5-5-4-4-4-4
[1111000110101000000000100100101110000000010010] = H(C,f2_1), 53 cells, 2l-2r
[1111000100101010000000001101010110001000000010] = H(C,f3_1), 53 cells, 3l-1r
H(C,f4_1) = H(D,f1_1)
[1111011000000001001110001001000000] = H(D,f1_1), 39 cells, 1l-0r -> destroy
[11110001101000000010100000101000000100000010] = H(D,f2_1), 53 cells, 2l-3r -> 3-4-5-5-4-4-5-4-4-4-4-4-5-6-5-5-5-5-4-4-4-4
[1111000100101000000010010101010000110010] = H(D,f3_1), 53 cells, 3l-2r
H(D,f4_1) = H(E,f1_1)
[1110100000111001011100001100010010] = H(E,f1_1), 39 cells, 1l-1r -> destroy
[1111000110001000010100010110000101001011100010010] = H(E,f2_1), 53 cells, 2l-3r -> 3-4-5-5-4-4-5-4-4-4-4-4-5-6-5-5-5-5-4-4-4-4
[11110001001001000110010110010010] = H(E,f3_1), 53 cells, 3l-2r
H(E,f4_1) = H(F,f1_1)
[1111010100100010101110000110000010] = H(F,f1_1), 39 cells, 1l-1r -> destroy
[111100011111101011110001001001000] = H(F,f2_1), 39 cells, 2l-0r
[111100010010000000110000001001010100100010010] = H(F,f3_1), 53 cells, 3l-3r -> 3-4-5-5-4-4-5-4-4-4-4-4-5-6-5-5-5-5-4-4-4-4
[11000000010100001011010101010010] = H(F,f4_1), 39 cells, 0l-2r
[11110000011100001110001100010010] = H(G,f1_1), 39 cells, 1l-1r -> destroy
[11110001100001000000001001
```

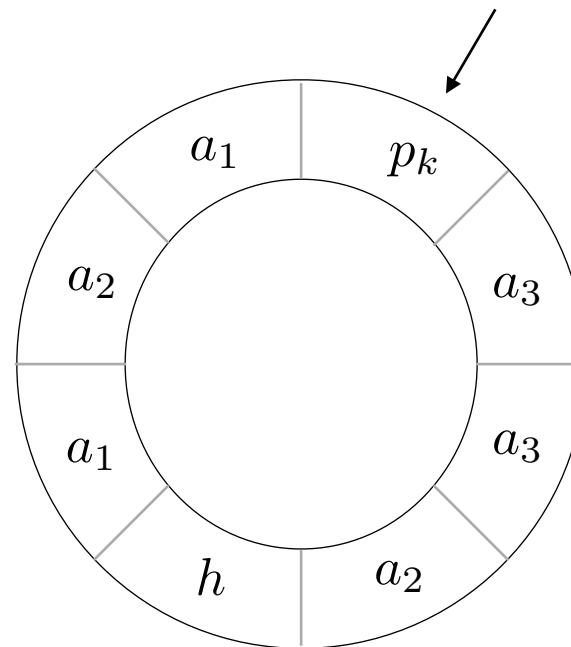
Particles as strings are implemented in OSXLCAU21 and CAViewer software



Circular computation

We have two important previous results in computer science theory to think about of circular computation. Arbib presents a circular Turing machine in 1962 and Kudlek and Rogozhin presents circular Post machines in 2001.

- a_i, h : symbols
- h : the limit of the type
- p_k : state
- $\rightarrow$: head



Important features: (1) label and limiting the end of the type, (2) the movement is turning the type, (3) the type can introduce new squares.

Symbol super colliders (Tommaso Toffoli, 2002)

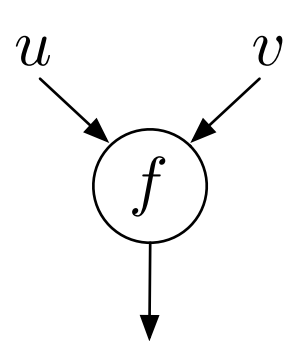
In 2002, Toffoli exposed the concept of “symbol super collider”. To map Toffoli's supercollider onto a one-dimensional cellular automata we use the notion of an idealized particle $p \in \mathbb{Z}^+$ (without energy and potential energy). The particle p is represented by a binary string of cell states. Typically, we can find all types of particles manifest in cellular automata particles, including positive p^+ , negative p^- , and neutral p^0 displacements, and also composite particles assembled from elementary particles.

(a) $f(u, v)$ is a product of one collision

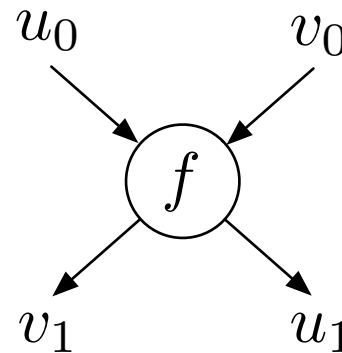
(b) $f(u, v) = u + v$ union

(c) $f_i(u, v) \mapsto (u, v)$ identity

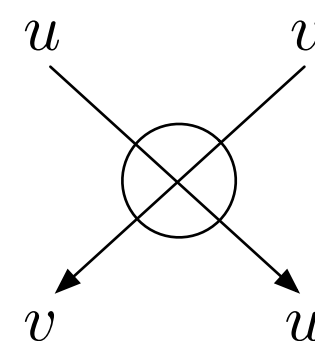
(d) $f_r(u, v) \mapsto (v, u)$ reflection



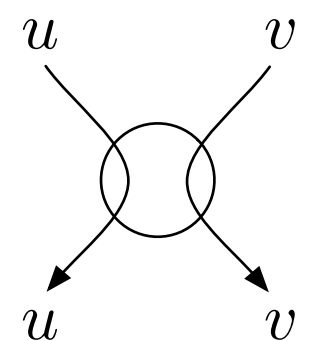
(a)



(b)

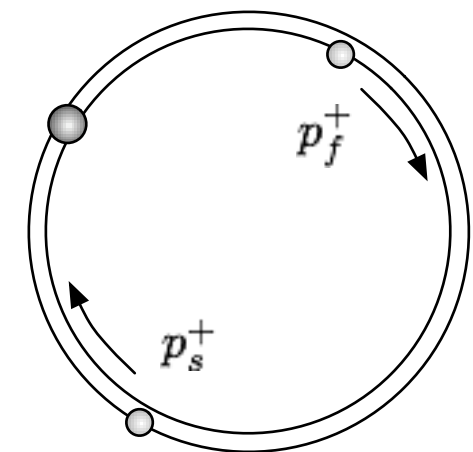
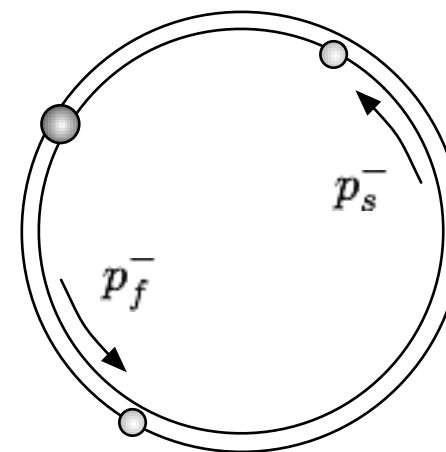
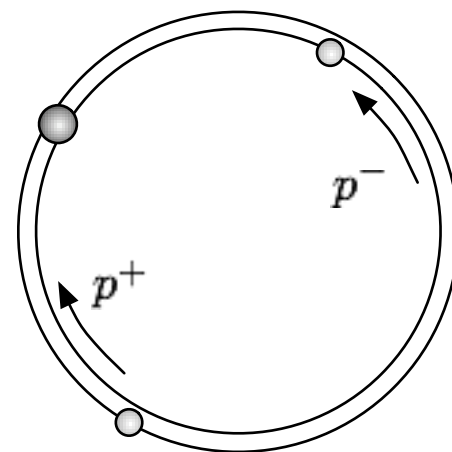


(c)



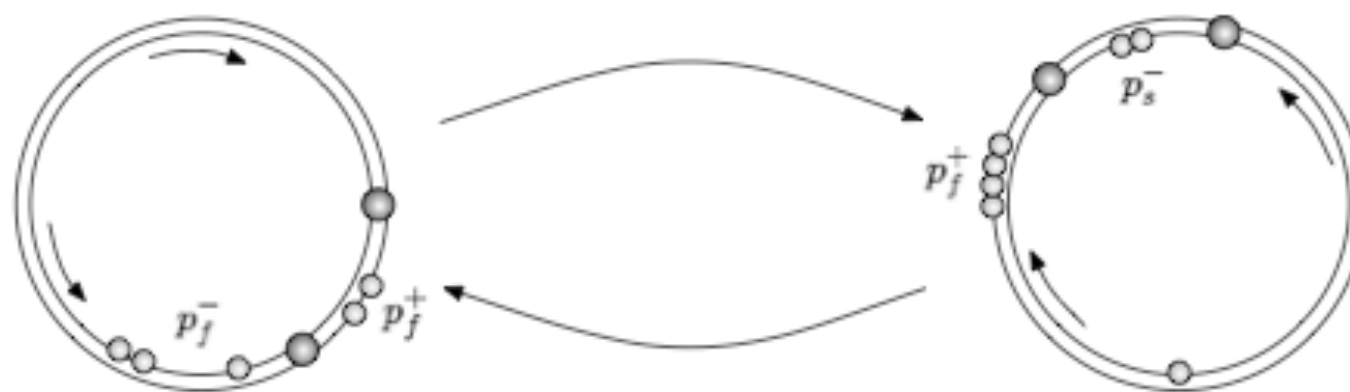
(d)

Schemes of ballistic collisions between particles evolving in *cyclotrons*. Gray circles represent the contact point of collision.



Symbol super colliders

Transition between two beam routing synchronizing multiple reactions. When the first set of collisions are done a new beam routing is defined with other particles, so that when the second set of collisions is done then one returns to the initial condition of the first beam, constructing a meta-glider or mesh in Rule 110.



In this way, we can design more complex constructions synchronizing multiple collisions with a diversity of speeds and phases on different particles. Figure displays a more sophisticated beam routing design, connecting two of beams and then creating a new beam routing diagram where edges represent a change of particles and collisions contact point on ECA Rule 110. In such a transition, a number of new particles emerge and collide to return to the first beam, thus oscillating between two beam routing forever.

$$p_A^+, p_A^+ \leftrightarrow p_B^-, p_B^-, p_B^-$$

changing to the set of particles (second beam routing):

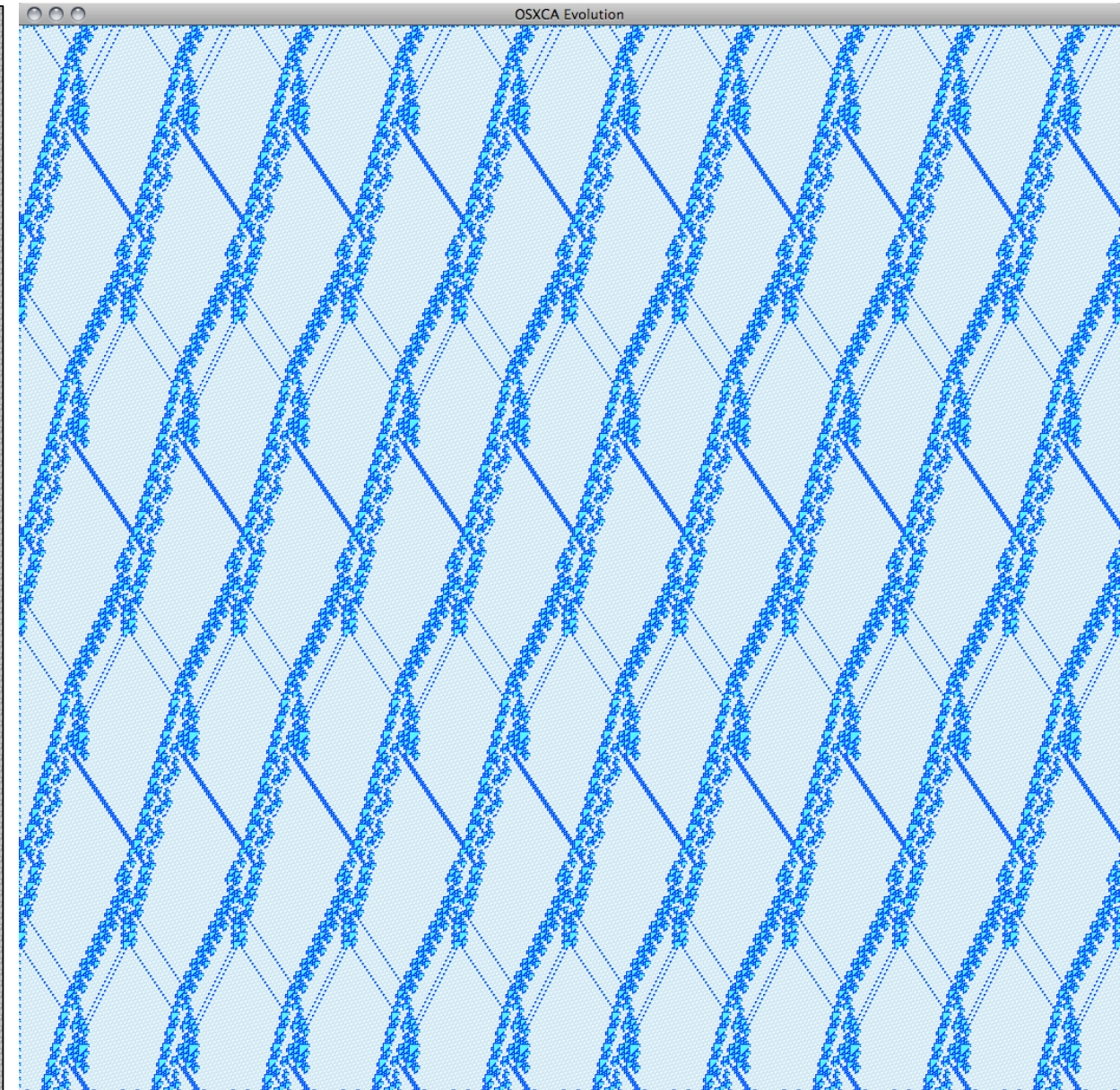
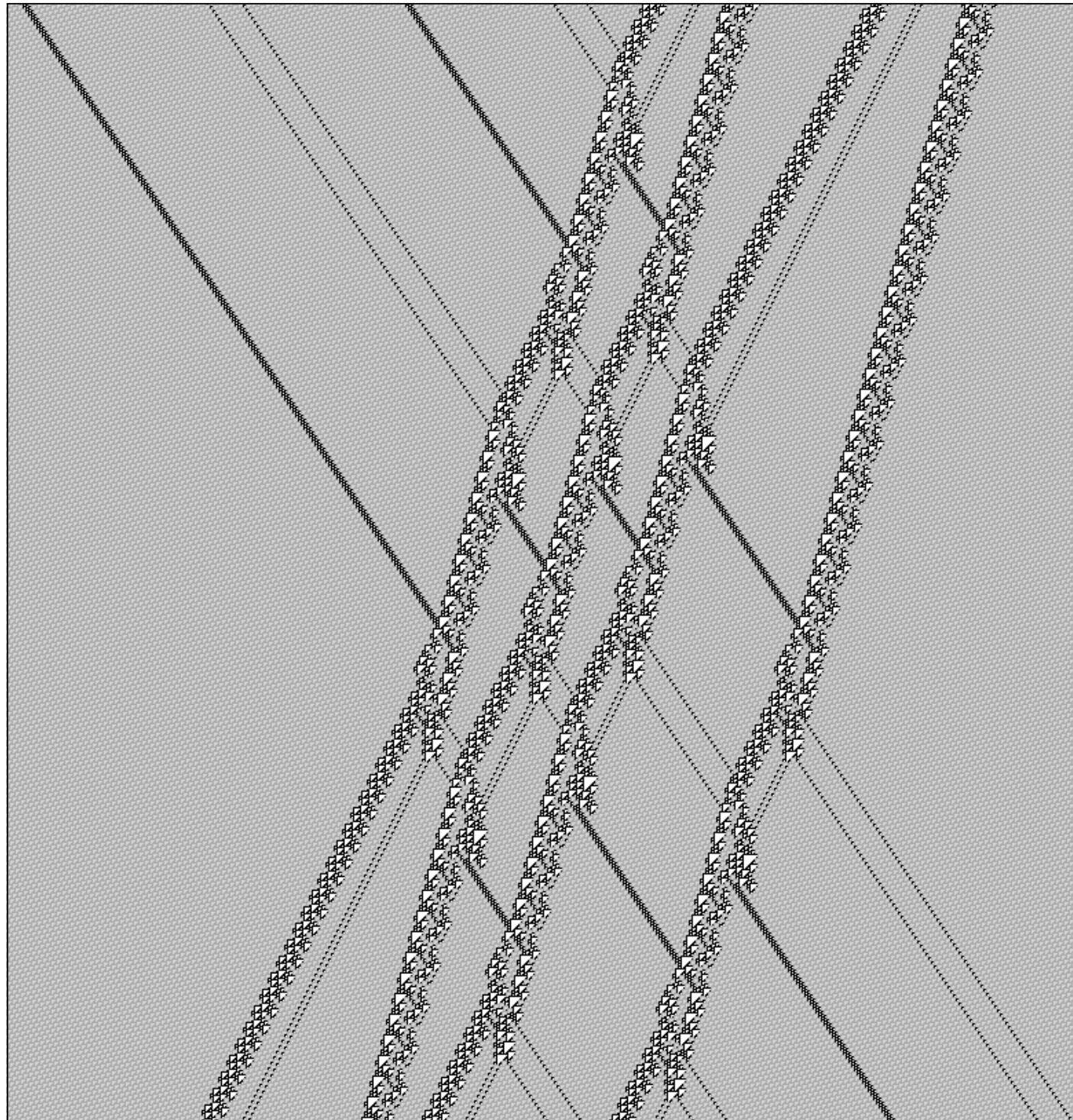
$$p_{A^4}^+ \leftrightarrow p_E^+, p_E^+$$

defining two beam routing connected by a transition of collisions as:

$$(p_A^+, p_A^+ \leftrightarrow p_B^-, p_B^-, p_B^-) \rightarrow (p_{A^4}^+ \leftrightarrow p_E^+, p_E^+), \text{ and}$$

$$(p_{A^4}^+ \leftrightarrow p_E^+, p_E^+) \rightarrow (p_A^+, p_A^+ \leftrightarrow p_B^-, p_B^-, p_B^-).$$

Symbol super colliders



$$(A(f_3-1)-e-A(f_1-1)-e-\overline{B}(C, f_1-1)-e-2B(f_4-1))^*$$

We use “Discrete Dynamics Lab” (DDLab). A free software created by Andrew Wuensche from 1992 to work with circular simulations. <http://www.ddlab.com/>

HOME PAGE

Discrete Dynamics Lab

Tools for researching Cellular Automata, Random Boolean Networks, multi-value Discrete Dynamical Networks, and beyond

Andy Wuensche
andy AT dddlab DOT org

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Visiting Professor
Unconventional Computing Lab
University of the West of England

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pencil drawing from the very early days
before automatic computer drawing was perfected

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[José Manuel Gómez Soto](#)
[Universal Computation in 2D CA](#)

ddlabz07: update November 2018
New options for investigating "[Pulsing dynamics in randomly wired glider cellular automata](#)" and "[The cellular automaton pulsing model](#)", for 2D 3-value totalistic CA. Updates include:

- Extending the entropy-density and density-return map plots to include a linked history.
- New methods to automatically recognise the wavelength/waveheight, and output/save data.
- Raspberry Pi executable is available ([readme](#)).

The DDLab screen showing pulsing dynamics. The entropy-density plot shows a linked-history (v3k7 g3).

(a) Value-density oscillations with time. 0=green, 1=brown, 2=black. (b) The density return-map scatter plot --- the density of each value at t (x-axis) against its density at $t+1$, plotted as colored dots, for about 33000 time-steps. (c) The same plot for just a few pulsing cycles, but linking successive dots giving a time-history. (v3k7 g1 [Spiral-rule](#))

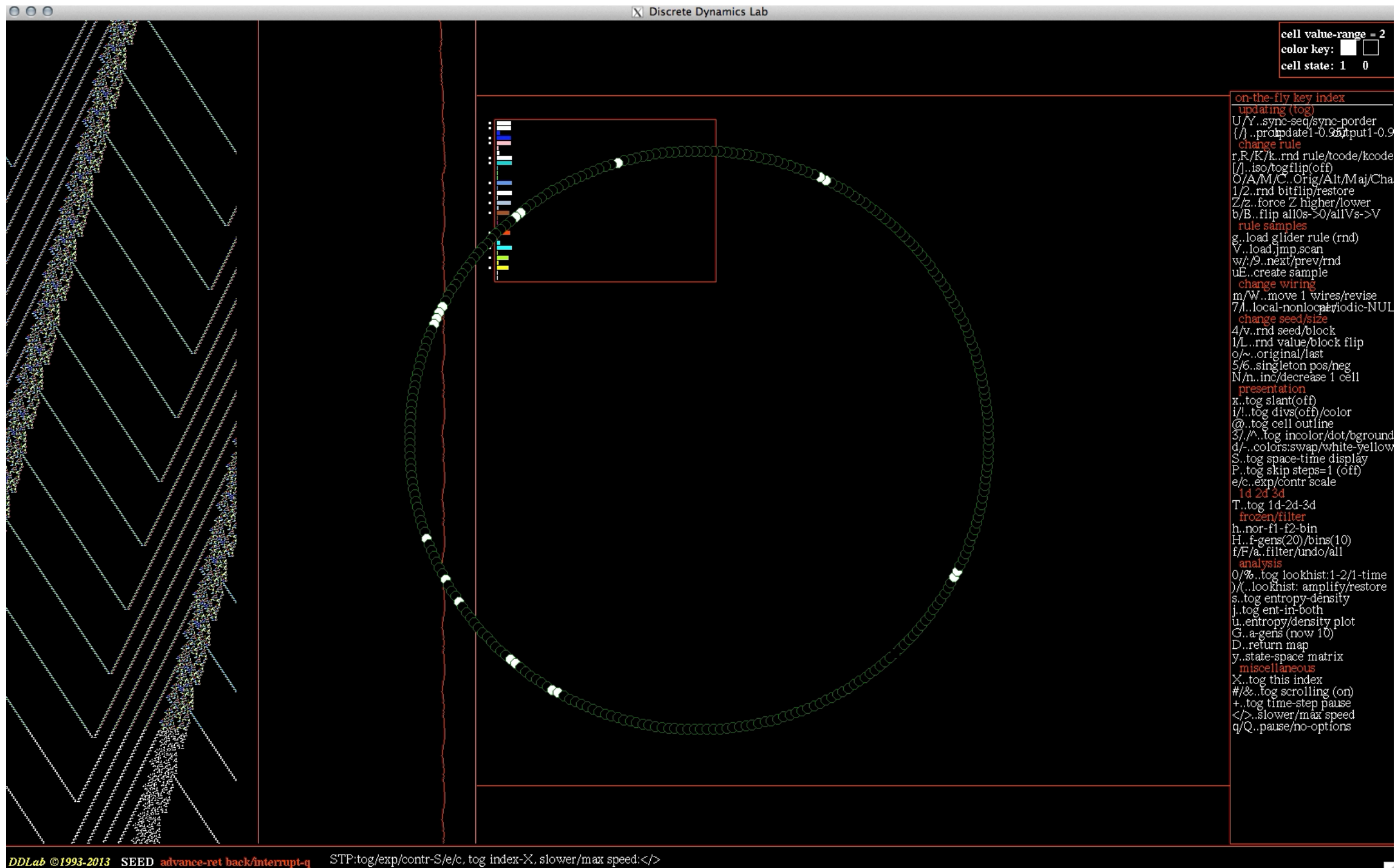
These latest updates are summarised [HERE](#) (feedback to: andy AT dddlab DOT org)

Documentation	Exploring Discrete Dynamics -- Second Edition -- The DDLab Manual The updated hyperref pdf which corresponds exactly to dddlabz07 Nov 2018. Click HERE to download (27M).
ddlab compiled Nov2018	compiled dddlabz07, for Linux, Raspberry Pi, Mac, Cygwin, DOS, and readme files.
ddlab code Nov2018	source code for dddlabz07, source code readme, and Makefiles
DDLab University of Sussex University of the West of England	As above from these mirror sites, compiles, code, and documentation, For the latest version, and an archive of all previous versions dating back to 1995.
	A directory of compiles, code, and documentation.

DDLlab evolving cellular automata as rings (cyclotrons)

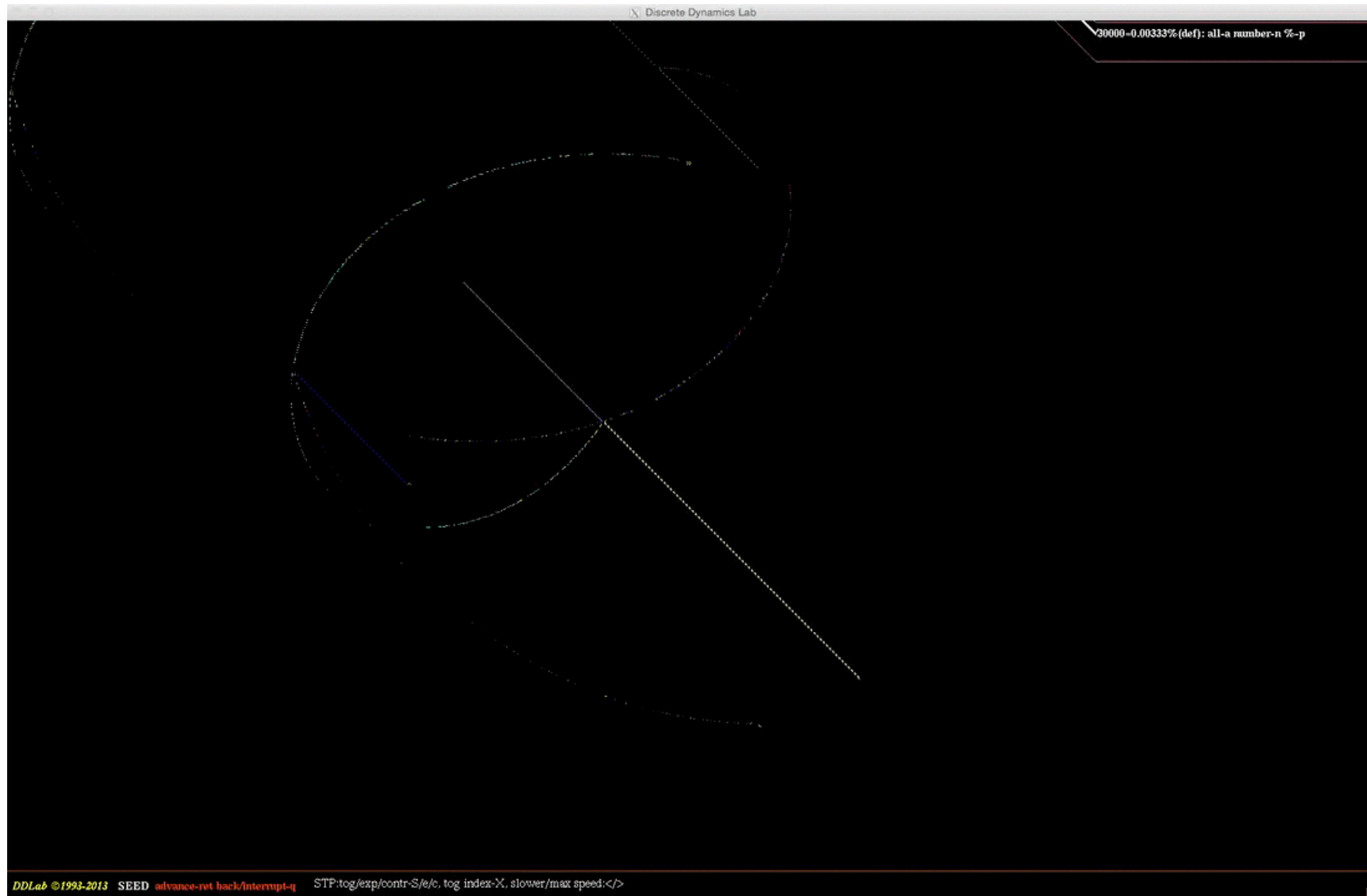
Cyclotrons are the first stage where we can see periodic collisions or simple dynamics of particles traveling around the ring.

A multiple collision between eight particles we can produce a glider gun in rule 110. In this simulation using DDLab, we can see the typical two dimensional representation and at the center the cyclotronic view, with the periodic background filtered.



Implementing a counter of positive integers by particle collisions in rule 54

This collider calculate the sequence of positive integers counting the number of T s in a stationary particle at the center by the number of collisions rule 54

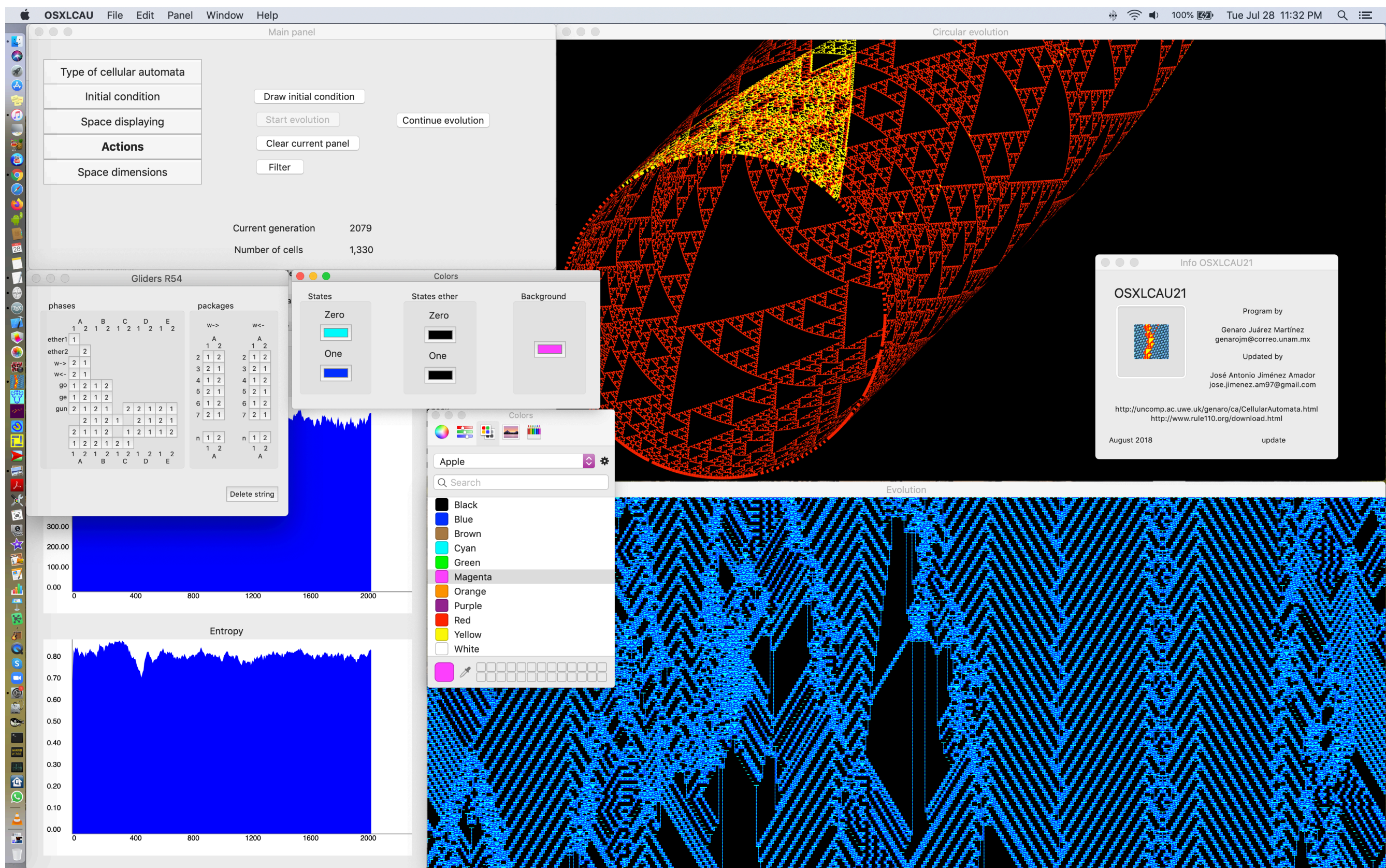


* Martinez, G.J., Adamatzky, A. & Stephens, C.R. (2011) **Cellular automaton supercolliders**, *International Journal of Modern Physics C* 22(4):419-439.

* Martinez, G.J., Adamatzky, A. & McIntosh, H. (2012) **Computing on rings**. In: *A computable Universe: Understanding and Exploring Nature as Computation*, H. Zenil (ed.), World Scientific Press, chapter 14, pages 283--302.

Cellular Automata Viewer (CAViewer) is a free software for mac where you can simulate circular evolutions (developed in LCCOMP, ALIROB, UCL; Labs)

https://www.comunidad.escom.ipn.mx/genaro/Cellular_Automata_Repository/Software.html



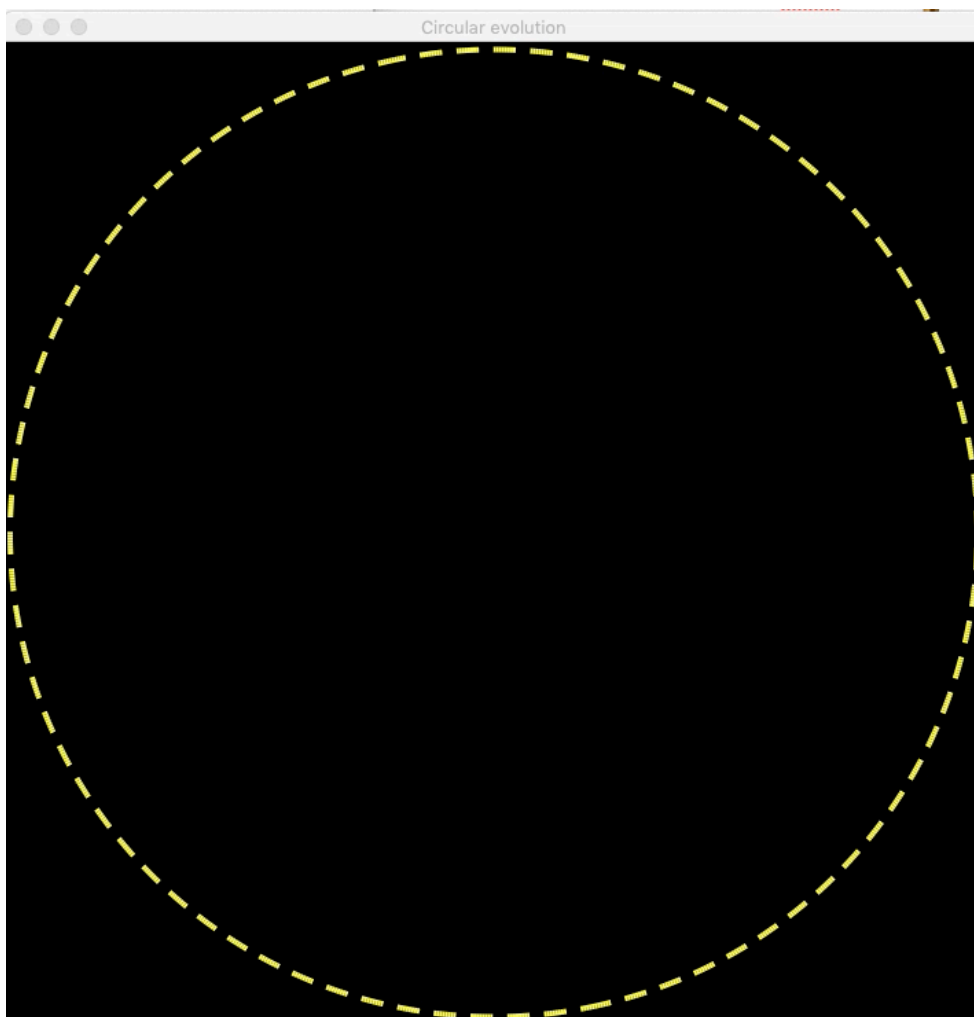
Construction of patterns by synchronization of multiple collisions in rule 54

Using a cyclotron we can design patterns with other views. This simulation starts with an initial condition of 3,214 cells codifying 216 particles in rule 54. The reaction is a triple collision controller by two basic interactions:

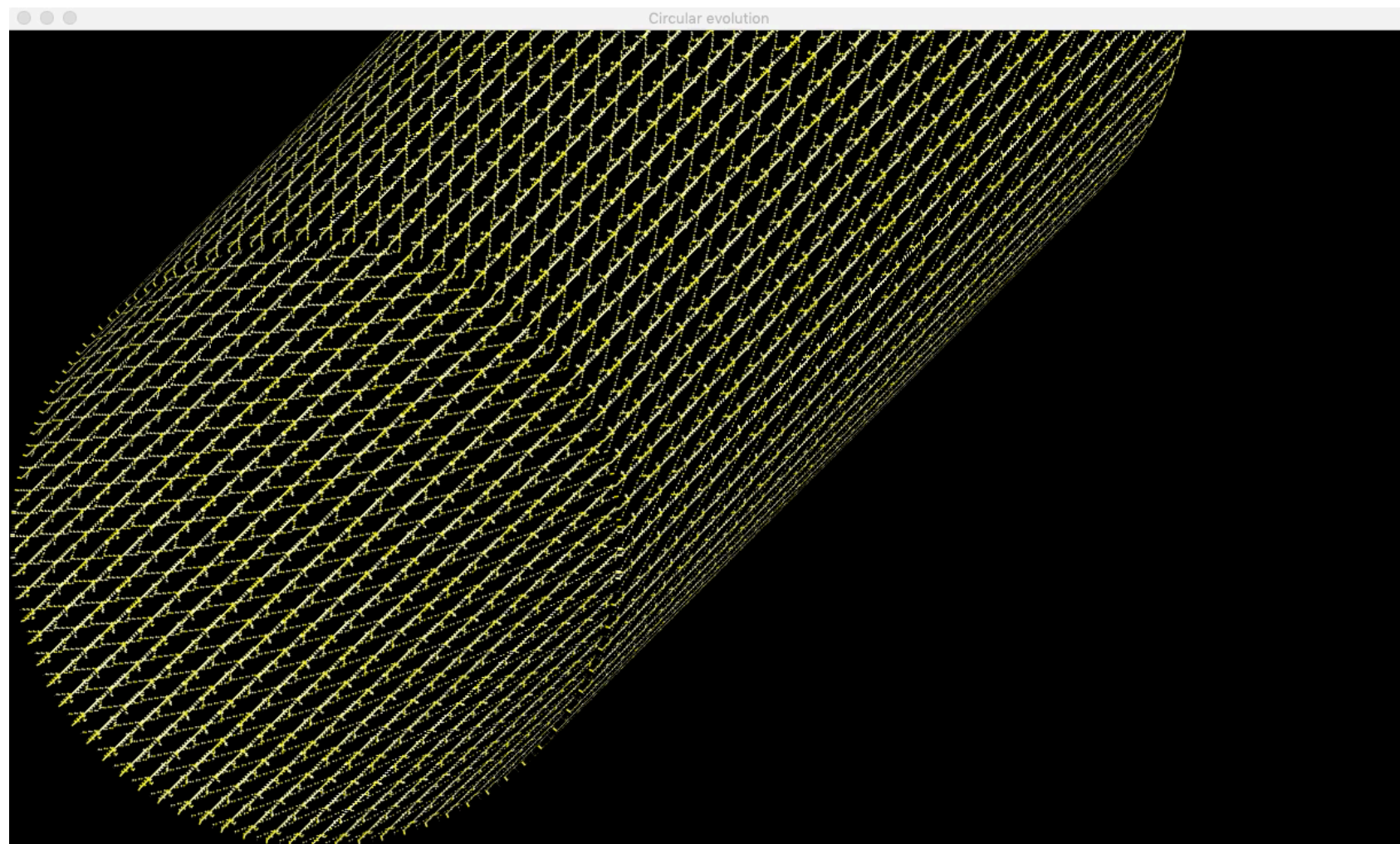
$$f(g_e, \overleftarrow{w}) \rightarrow \overleftarrow{w}, g_e, 2\overrightarrow{w} ; f(2\overrightarrow{w}, \overleftarrow{w}) \rightarrow \overleftarrow{w}$$

The reaction starts with a negative particle w colliding versus a stationary particle g yielding a negative particle plus two positive particles w . But the first negative particle finds these new pairs of positive particles and annihilate them.

Therefore, we can codify these particles with the next expression: $(\overleftarrow{w} - 2e - g_e(A, f_1) - 2e - \overleftarrow{w})^*$.

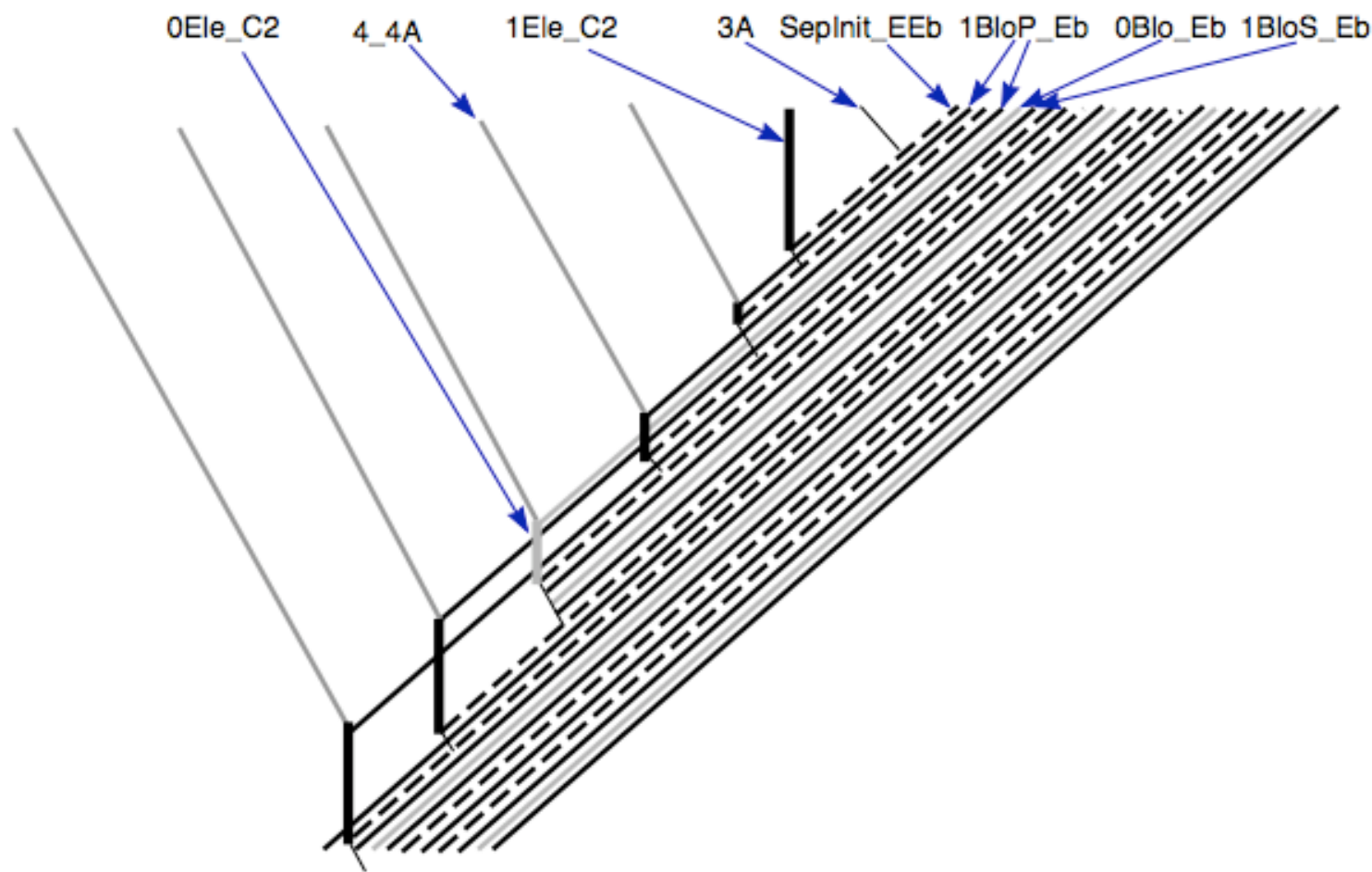


circular

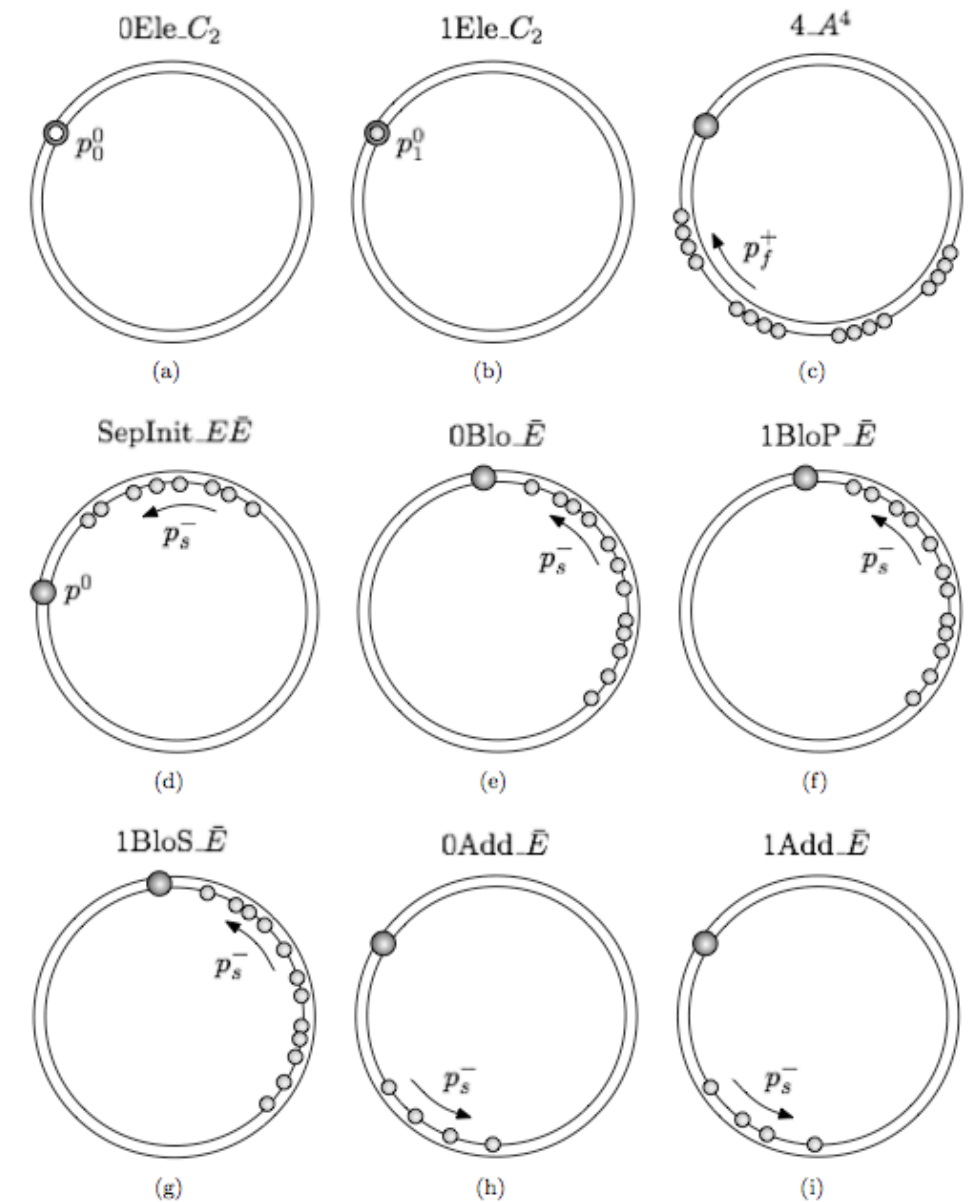


three-dimensional projection

Cyclic tag systems in a finite codification



A diagram of a cyclic tag system (CTS) working in rule 110



Beam routing codification representing package of particles which reproduces a CTS in rule 110

- Cook, M. (2004) **Universality in Elementary Cellular Automata**. Complex Systems 15(1), 1-40.
- Cook, M. (2008) **A Concrete View of Rule 110 Computation**. In: The Complexity of Simple Programs, T. Neary, D. Woods, A.K. Seda and N. Murphy (Eds.), 31-55.
- Wolfram, S. (2002) **A New Kind of Science**, Wolfram Media, Inc., Champaign, Illinois.
- Neary, T. & Woods, D. (2006) **P-completeness of cellular automaton Rule 110**. Lecture Notes in Computer Science 4051, 132-143.
- Martínez, G.J., McIntosh, H.V., Mora, J.C.S.T. & Vergara, S.V.C. (2011) **Reproducing the cyclic tag system developed by Matthew Cook with Rule 110 using the phases f1_1**, *Journal of Cellular Automata* 6(2-3), 121-161.

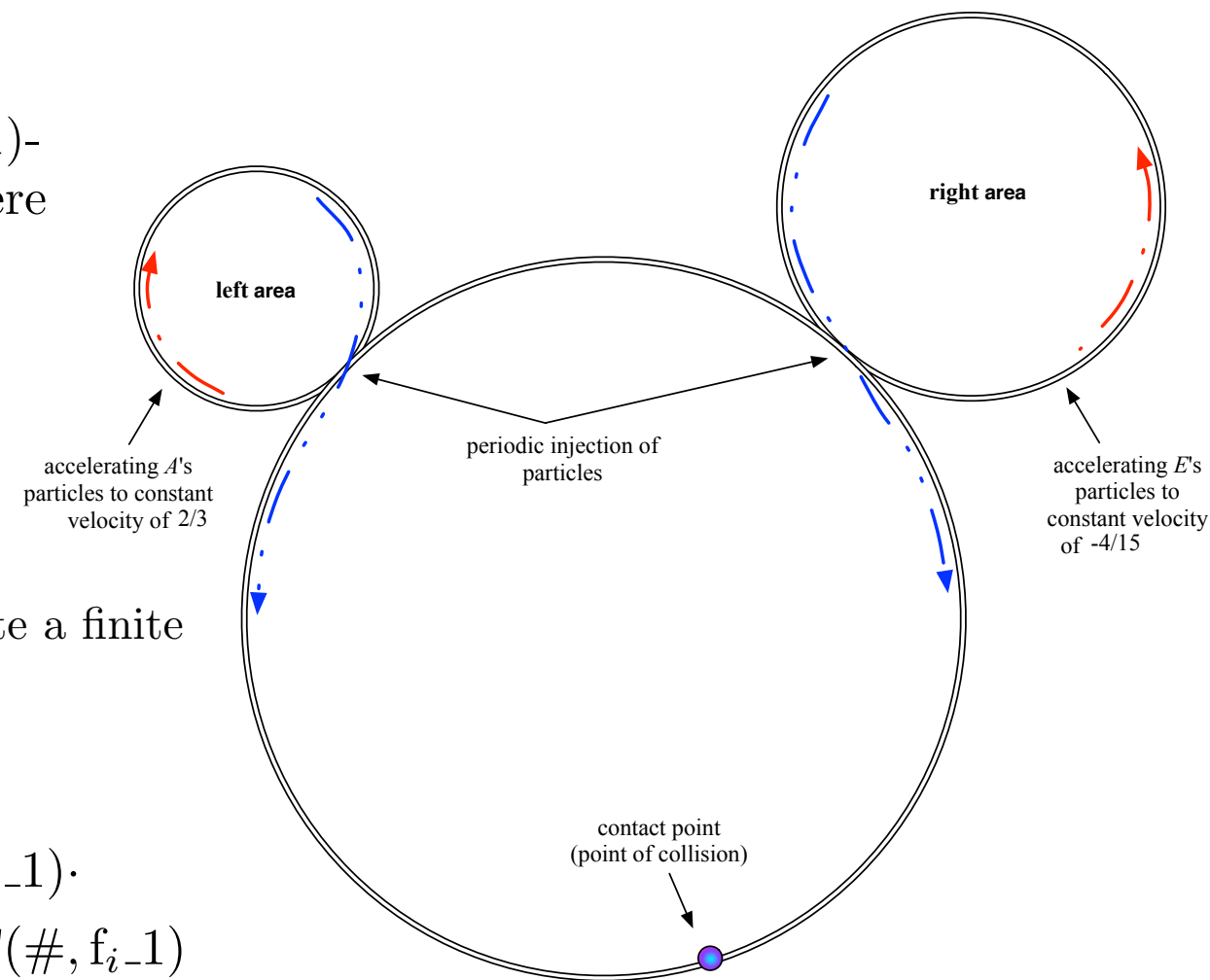
Cyclic tag systems in a finite codification as a collider

This way, the cyclic tag system working in rule 110 can be simplified as follows:

left: $\{649e-4_A^4(F_i)\}^*$, for $1 \leq i \leq 3$ in sequential order

center: $246e-1Ele_C2(A,f_1-1)-e-A^3(f_1-1)$

right: $\{\text{SepInit_}E\bar{E}(\#,f_i-1)-1\text{BloP_}\bar{E}(\#,f_i-1)-\text{SepInit_}E\bar{E}(\#,f_i-1)-1\text{BloP_}\bar{E}(\#,f_i-1)-0\text{Blo_}\bar{E}(\#,f_i-1)-1\text{BloS_}\bar{E}(\#,f_i-1)\}^*$ (where $1 \leq i \leq 4$ and $\#$ represents a particular phase).



A diagram of a cyclic tag system (CTS) working in rule 110

This way, we have that the string $w_{CTSR110}$ is a word able to simulate a finite state machine into a cellular automata collider.

$$w_{CTSR110} = (649e \cdot 4_A^4(F_i))^* \cdot (246e \cdot 1Ele_C2(A,f_1-1) \cdot e \cdot A^3(f_1-1)) \cdot (\text{SepInit_}E\bar{E}(\#,f_i-1) \cdot 1\text{BloP_}\bar{E}(\#,f_i-1) \cdot \text{SepInit_}E\bar{E}(\#,f_i-1) \cdot 1\text{BloP_}\bar{E}(\#,f_i-1) \cdot 0\text{Blo_}\bar{E}(\#,f_i-1) \cdot 1\text{BloS_}\bar{E}(\#,f_i-1))^*.$$

A Computation in a Cellular Automaton Collider Rule 110



Final remarks

Complex ECA rules with different capacities explored with cyclotrons.

rule	class	particle	particle ⁿ	slopes	gun	gun ⁿ	soliton	complex with memory	fractals
41	4	yes	no	+	no	no	no	yes	no
54	4	yes	no	-,+,s	yes	yes	yes	yes	no
106	4	yes	no	-	no	no	no	yes	no
110	4	yes	yes	-,+,s	yes	yes	yes	yes	no
22	3	yes	no	-,+	no	no	no	yes	yes
126	3	yes	no	-,+,s	no	no	no	yes	yes
26	2	yes	no	-,+	no	no	yes	yes	yes
62	2	yes	no	-,+	no	no	no	yes	no

- Martínez, G.J., Adamatzky, A., Hoffmann, R., Désérable, D. & Zelinka, I. (2019) **On Patterns and Dynamics of Rule 22 Cellular Automaton**. *Complex Systems* 28(2), 125-174.
- Martínez, G.J., Adamatzky, A. & Alonso-Sanz, R. (2013) **Designing Complex Dynamics in Cellular Automata with Memory**. *International Journal of Bifurcation and Chaos* 23(10), 1330035-131.

THE END

THANK YOU FOR YOUR KIND ATTENTION

Cellular Automata Repository

https://www.comunidad.escom.ipn.mx/genaro/CA_repository.html

Complex Cellular Automata Repository

https://www.comunidad.escom.ipn.mx/genaro/Complex_CA_repository.html

Cellular Automata Software

https://www.comunidad.escom.ipn.mx/genaro/Cellular_Automata_Repository/Software.html